Day 60 **S Tier 10-a-day: Model Solutions**

1 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 24 ÷ 2 = 12 cm
$OM^2 = OA^2 - AM^2 = 13^2 - 12^2 = 25$
OM = 5 cm

(b) sin(angle AOM) = $\frac{12}{13}$
Angle AOM = 67.38°, so angle AOB = 2 × 67.38 = 134.8°
Answer: (a) 5 cm (b) 134.8°

2 Algebraic fractions: adding and subtracting

Show that $\frac{x+3}{2x-1} - \frac{6}{x+5}$ can be written as $\frac{x^2-4x+21}{(2x-1)(x+5)}$

The common denominator is $(2x-1)(x+5)$
$$\frac{x+3}{2x-1} - \frac{6}{x+5} = \frac{(x+3)(x+5)-6(2x-1)}{(2x-1)(x+5)}$$
Expand the numerator: $x^2 - 4x + 21$
$$\frac{x^2-4x+21}{(2x-1)(x+5)}$$
Answer: $\mathbf{\frac{x^2-4x+21}{(2x-1)(x+5)}}$

3 Distance-time graphs

The graph shows Callum's cycle ride to a café and back.
(a) Work out Callum's speed on the way back.
(b) Work out Callum's average speed for the whole ride, including the stop.

(a) Return: 45 km in 4.5 hours, so speed = 45 ÷ 4.5 = 10 km/h
(b) Total distance = 2 × 45 = 90 km, total time = 7.5 hours
Average speed = 90 ÷ 7.5 = 12 km/h
Answer: (a) 10 km/h (b) 12 km/h

4 Fraction problems

A fuel tank is $\frac{3}{4}$ full.

18 more litres of petrol are added and the fuel tank is then $\frac{4}{5}$ full.

Work out the capacity of the fuel tank.

Fraction added = $\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

$\frac{1}{20}$ of the capacity is 18 litres

Capacity = $18 \div \frac{1}{20} = 360$

Answer: 360 litres

Day 60 **S Tier 10-a-day: Model Solutions**

5 Area of a triangle using sine

In triangle PQR, PQ = 18 cm and angle PQR = 150°.
The area of the triangle is 45 cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$45 = \frac{1}{2} \times 18 \times QR \times \sin 150°$

$\sin 150° = \frac{1}{2}$, so $45 = \frac{9}{2} \times QR$

$QR = 45 \div \frac{9}{2} = 10$ cm

Answer: 10 cm

6 Reverse percentage growth and decay

The population of a town grows by 10% each year.
After 4 years the population is 24,889.
Work out the population 4 years ago. (Calculator)

Yearly multiplier = 1.1
Original $\times 1.1^4 = 24{,}889$
Original $= 24{,}889 \div 1.1^4 = 17{,}000$
Answer: 17,000

7 Laws of indices

Write $\frac{4^{n+3} \times 8^{n+3}}{4^n}$ in the form 2^{an+c}
where a and c are integers.

$4 = 2^2$, $8 = 2^3$, $4 = 2^2$
$4^{n+3} = 2^{2(n+3)}$, $8^{n+3} = 2^{3(n+3)}$, $4^n = 2^{2n}$
Exponent: $2(n + 3) + 3(n + 3) - 2n = 3n + 15$
Answer: a = 3, c = 15

8 Equal roots and real roots

Show that the equation $x^2 + kx - 6 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = k^2 - 4 \times 1 \times (-6)$
$= k^2 + 24$
k^2 is never negative, so $k^2 + 24$ is always at least 24, which is greater than 0
So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions
Answer: Shown

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 8 cm, top radius 6 cm and base radius 18 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+8} = \frac{6}{18}$
18 h = 6(h + 8), so 12 h = 48 and h = 4
Large cone: $\frac{1}{3} \times \pi \times 18^2 \times 12 = 1296\pi$
Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 4 = 48\pi$
Frustum = $1296\pi - 48\pi = 1248\pi$ cm^3
Answer: 1248π cm^3

Day 60 **S Tier 10-a-day: Model Solutions**

10 Composite functions: fg and gf

$f(x) = 3x + 5$ and $g(x) = x^2 - 2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 2) = 3x^2 - 1$
$gf(x) = g(3x + 5) = (3x + 5)^2 - 2 = 9x^2 + 30x + 23$
Set them equal and rearrange: $6x^2 + 30x + 24 = 0$
Divide by 6 and factorise: $(x + 1)(x + 4) = 0$
$x = -4$ or $x = -1$
Answer: $x = -4$ or $x = -1$