Day 59 **S Tier 10-a-day: Model Solutions**

1 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 81}{x^2 + 5x} \div \frac{x^2 - 7x - 18}{x}$

Factorise: $\frac{(x - 9)(x + 9)}{x(x + 5)} \div \frac{(x + 2)(x - 9)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x - 9)(x + 9)}{x(x + 5)} \times \frac{x}{(x + 2)(x - 9)}$

Cancel $(x - 9)$ and x

$= \frac{(x + 9)}{(x + 2)(x + 5)}$

Answer: $\mathbf{\frac{(x + 9)}{(x + 2)(x + 5)}}$

2 Percentage change

The price of a phone is increased by 60%.
The new price is then decreased by 25%.
Work out the overall percentage change in the price.

Multipliers: 1.6 and 0.75
Overall multiplier = $1.6 \times 0.75 = 1.2$
$1.2 - 1 = 0.2$, so an overall increase of 20%
Answer: 20% increase

3 Arc length and sector area

A sector of a circle has radius 3 cm and angle 240°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{240}{360}$

Arc length = $\frac{240}{360} \times 2 \times \pi \times 3 = 4\pi$ cm

(b) Area = $\frac{240}{360} \times \pi \times 3^2 = 6\pi$ cm^2

Answer: (a) 4π cm (b) 6π cm^2

4 Gradient of a chord

A and B are points on the curve $y = x^3 + 5x$.
The x coordinate of A is -3 and the x coordinate of B is -1.
Work out the gradient of the line AB.

At A: $y = -42$. At B: $y = -6$
A is $(-3, -42)$ and B is $(-1, -6)$

Gradient = $\frac{-6 - (-42)}{-1 - (-3)} = \frac{36}{2} = 18$

Answer: 18

5 Speed-time graphs

The graph shows the speed of a cyclist during 12 seconds.
(a) Work out the acceleration during the first 3 seconds.
(b) Work out the average speed over the whole 12 seconds.

(a) Acceleration = change in speed ÷ time = $(16 - 4) \div 3 = 4$ m/s^2
(b) Distance = area under the graph

Trapezium: $\frac{4 + 16}{2} \times 3 = 30$; rectangle: $16 \times 6 = 96$; triangle: $\frac{1}{2} \times 3 \times 16 = 24$

Total = 150 m, so average speed = $150 \div 12 = 12.5$ m/s

Answer: (a) 4 m/s^2 (b) 12.5 m/s

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6 Combined transformations

The point P(3, 1) is reflected in the x-axis.
The image is then rotated 180° about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, −1)
After the second transformation: P'' = (−3, 1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (x, −y) → (−x, y)
The single mapping is (x, y) → (−x, y)
This is a reflection in the y-axis
Answer: (a) (−3, 1) (b) Reflection in the y-axis

7 Line of best fit

The scatter graph shows hours of revision (x) and test mark (y).
The line of best fit passes through (2, 15) and (8, 33).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{33 - 15}{8 - 2} = \frac{18}{6} = 3$
y = 3x + c: using (2, 15), 15 = 3 × 2 + c, so c = 9
y = 3x + 9
(b) For each extra hour of revision, the test mark goes up by 3
Answer: (a) y = 3x + 9 (b) For each extra hour of revision, the test mark goes up by 3

8 Area and quadratic equations

The diagram shows a rectangle of length (x − 1) cm, width (2x − 3) cm and area 28 cm^2.
(a) Show that $2x^2 - 5x - 25 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) (x − 1)(2x − 3) = 28
Expand: $2x^2 - 5x + 3 = 28$, so $2x^2 - 5x - 25 = 0$
(b) Factorise: (x − 5)(2x + 5) = 0
x = 5 ($x = -\frac{5}{2}$ is not possible as a length would be negative)
Length = 4 cm and width = 7 cm
Diagonal = $\sqrt{4^2 + 7^2} = \sqrt{65}$ = 8.06 cm (3 s.f.)
Answer: (b) 8.06 cm

9 Exponential growth and decay

A population of rabbits starts at 3,000 and grows by 20% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.2, double is 6,000
After 3 years: $3000 \times 1.2^3 = 5184.0$
After 4 years: $3000 \times 1.2^4 = 6220.8$
Answer: 4 years

10 Exact lengths and areas

A square has sides of length $(5 + \sqrt{6})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{6}$

Area = $(5 + \sqrt{6})^2$
$= 25 + 10\sqrt{6} + 6$
$= 31 + 10\sqrt{6}$
Answer: $31 + 10\sqrt{6}$ cm^2