Day 58 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.53, 0.6, $\frac{3}{8}$, $\frac{6}{11}$

Convert each to a decimal: 0.53 = 0.53, 0.6 = 0.6, $\frac{3}{8}$ = 0.375, $\frac{6}{11}$ = 0.545

In order: $\frac{3}{8}$, 0.53, $\frac{6}{11}$, 0.6

Answer: $\frac{3}{8}$, 0.53, $\frac{6}{11}$, 0.6

2 Distance-time graphs

The graph shows Kira's cycle ride to a café and back.

(a) Work out Kira's speed on the way back.

(b) Work out Kira's average speed for the whole ride, including the stop.

(a) Return: 30 km in 2.5 hours, so speed = 30 ÷ 2.5 = 12 km/h

(b) Total distance = 2 × 30 = 60 km, total time = 5 hours

Average speed = 60 ÷ 5 = 12 km/h

Answer: (a) 12 km/h (b) 12 km/h

3 Simplifying surds

Write $\sqrt{27} + \sqrt{48} - \sqrt{3} + \sqrt{108}$ in the form $a\sqrt{3}$ where a is an integer.

$\sqrt{27} = 3\sqrt{3}$; $\sqrt{48} = 4\sqrt{3}$; $\sqrt{108} = 6\sqrt{3}$

$(3 + 4 - 1 + 6)\sqrt{3} = 12\sqrt{3}$

Answer: $12\sqrt{3}$

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle APB = 80°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)

Angles in quadrilateral OAPB: 360 − 90 − 90 − 80 = 100°

(b) The angle at the circumference is half the angle at the centre: 100 ÷ 2 = 50°

Answer: (a) 100° (b) 50°

5 Quadratic sequences

Here are the first five terms of a quadratic sequence.

11, 19, 31, 47, 67

Find an expression for the nth term.

First differences: 8, 12, 16, 20

Second differences: 4, 4, 4, so a = 4 ÷ 2 = 2

Subtract $2n^2$ from each term: 9, 11, 13, 15, 17

The remaining linear sequence has nth term 2n + 7

nth term = $2n^2 + 2n + 7$

Answer: $2n^2 + 2n + 7$

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6 Three-part ratios

Isla, Evan and Beth share some sweets.
Isla : Evan = 6 : 3
Evan : Beth = 4 : 6
(a) Work out the ratio Isla : Evan : Beth.
(b) They share 63 sweets altogether. How many sweets does Evan get?

(a) Make the Evan part the same: 6 : 3 = 24 : 12 and 4 : 6 = 12 : 18
Isla : Evan : Beth = 24 : 12 : 18 = 4 : 2 : 3
(b) Total parts = 9, so 1 part = 63 ÷ 9 = 7
Evan gets 2 × 7 = 14 sweets
Answer: (a) 4 : 2 : 3 (b) 14 sweets

7 Exact trigonometric ratios from a given ratio

x is an obtuse angle and $\sin x = \frac{10}{13}$.
(a) Find the exact value of cos x.
(b) Find the exact value of tan x. Give each answer as a fraction or a surd. Do not use a calculator.

(a) $\cos^2 x = 1 - (\frac{10}{13})^2 = \frac{69}{169}$

x is obtuse, so cos x is negative: $\cos x = -\frac{\sqrt{69}}{13}$

(b) $\tan x = \frac{\sin x}{\cos x} = \frac{10}{13} \div (-\frac{\sqrt{69}}{13}) = -\frac{10\sqrt{69}}{69}$

Answer: (a) $-\frac{\sqrt{69}}{13}$ (b) $-\frac{10\sqrt{69}}{69}$

8 Equation of a circle

A and B are the points (−5, −4) and (−15, 2).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{-5 + (-15)}{2}, \frac{-4 + 2}{2}) = (-10, -1)$
$r^2 = (-5 - (-10))^2 + (-4 - (-1))^2 = 25 + 9 = 34$
$(x + 10)^2 + (y + 1)^2 = 34$
Answer: $(x + 10)^2 + (y + 1)^2 = 34$

9 Exact lengths and areas

A square has sides of length $(6 + 2\sqrt{2})$ cm.
Work out the area of the square.
Give your answer in the form $a + b\sqrt{2}$

Area $= (6 + 2\sqrt{2})^2$
$= 36 + 24\sqrt{2} + 8$
$= 44 + 24\sqrt{2}$
Answer: $44 + 24\sqrt{2}$ cm^2

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10 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 30 ÷ 2 = 15 cm

$OA^2 = OM^2 + AM^2 = 8^2 + 15^2 = 289$

OA = 17 cm

(b) sin(angle AOM) = $\frac{15}{17}$

Angle AOM = 61.93°, so angle AOB = 2 × 61.93 = 123.9°

Answer: (a) 17 cm (b) 123.9°