Day 57 **S Tier 10-a-day: Model Solutions**

1 Capture and recapture

50 deer in a forest are caught, marked and put back.
Later, a second sample of 50 deer is caught. 4 of these are marked.
(a) Work out an estimate for the number of deer in a forest.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 4 out of 50

This should match the proportion marked in the whole population: $\frac{4}{50} = \frac{50}{N}$

N = 50 × 50 ÷ 4 = 625

(b) No deer are born, die or leave between the samples, and the deer mix evenly

Answer: (a) 625 (b) The population stays the same between samples

2 Reverse percentages

The number of members of a club is increased by 10% in one year.
The next year the number of members is decreased by 25%.
The next year the number of members is increased by 5%.
There are now 693 members.
How many members were there to begin with?

Multipliers: 1.1, 0.75, 1.05
Overall multiplier = 1.1 × 0.75 × 1.05 = 0.86625
Original = final ÷ multiplier = 693 ÷ 0.86625
= 800

Answer: 800

3 Solving trigonometric equations

Solve 5 tan x − 7 = 0 for 0° ≤ x ≤ 360°. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = \frac{7}{5}$

Related acute angle: $\tan^{-1}(\frac{7}{5}) = 54.5°$

tangent is positive in the 1st and 3rd quadrants, so x = 54.5° and x = 180° + 54.5°
x = 54.5° and x = 234.5°

Answer: x = 54.5° or x = 234.5°

4 Percentage change and proportion

The brightness of a lamp is inversely proportional to the square of the distance from the lamp.
The distance from the lamp is increased by 100%.
Work out the percentage change in the brightness of a lamp.

The distance from the lamp is multiplied by 2

The multiplier for the brightness of a lamp is $1 \div 2^2 = 0.25$
0.25 is a decrease of 75%

Answer: 75% decrease

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5 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-6-0}{0-(-3)} = -2$

Perpendicular gradient = $-1 \div (-2) = \frac{1}{2}$

M passes through B(−3, 0): $0 = \frac{1}{2} \times (-3) + c$, so $c = \frac{3}{2}$

$y = \frac{1}{2}x + \frac{3}{2}$

Answer: $\mathbf{y = \frac{1}{2}x + \frac{3}{2}}$

6 Vectors in a triangle

OAB is a triangle. OA = **a** and OB = **b**.
M is on OA with OM : MA = 3 : 1. N is on AB with AN : NB = 1 : 3.
(a) Find ON in terms of **a** and **b**.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{4}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a}) = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

(b) OM = $\frac{3}{4}\mathbf{a}$

MN = ON − OM = $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b} - \frac{3}{4}\mathbf{a} = \frac{1}{4}\mathbf{b}$

MN = $\frac{1}{4}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\mathbf{\frac{3}{4}a + \frac{1}{4}b}$ **(b) MN =** $\mathbf{\frac{1}{4}b}$**, parallel to OB**

7 Winning a match

Beth and Callum play a match. The first to win 2 table tennis games wins the match.

The probability that Beth wins any one of the table tennis games is $\frac{1}{3}$.

Work out the probability that Beth wins the match.

Beth wins by WW (2 games) or WLW or LWW (3 games), where W means Beth wins that game

P(WW) = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

P(WLW) = P(LWW) = $\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$

Total = $\frac{1}{9} + 2 \times \frac{2}{27} = \frac{7}{27}$

Answer: $\mathbf{\frac{7}{27}}$

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8 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 10$

$x + y = 4$

From the second equation, $y = -x + 4$

Substitute into the first equation: $x^2 + (-x + 4)^2 = 10$

Expand: $2x^2 - 8x + 6 = 0$

Divide by 2: $x^2 - 4x + 3 = 0$

Factorise: $(x - 1)(x - 3) = 0$, so $x = 1$ or $x = 3$

$y = -x + 4$ gives $y = 3$ and $y = 1$

Answer: x = 1, y = 3 and x = 3, y = 1

9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.

OP = $-3\mathbf{a} - \mathbf{b}$, OQ = $-2\mathbf{a} + 4\mathbf{b}$ and OR = $2\mathbf{a} + 24\mathbf{b}$.

(a) Show that P, Q and R lie on a straight line.

(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $\mathbf{a} + 5\mathbf{b}$

QR = OR − OQ = $4\mathbf{a} + 20\mathbf{b}$

QR = 4 PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 1 : 4

Answer: (b) 1 : 4

10 Proof with indices

Show that $5^{n+2} - 5^n$ is a multiple of 24 for all integer values of n.

$5^{n+2} = 5^n \times 5^2$

$5^{n+2} - 5^n = 5^n(5^2 - 1)$

$5^2 - 1 = 24$

$= 24 \times 5^n$, which is a multiple of 24

Answer: 24×5^n