Day 56 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $a^2 - 3bc$
when $a = 5$, $b = -4$ and $c = 1$.

Substitute the values: $5^2 - 3 \times (-4) \times 1$
$= 25 - (-12)$
$= 37$
Answer: 37

2 Rationalising the denominator

Rationalise the denominator of $\frac{19}{2\sqrt{7}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{7}$:
$\frac{19}{2\sqrt{7}} = \frac{19\sqrt{7}}{14}$
Answer: $\frac{19\sqrt{7}}{14}$

3 Weighted percentages

The ratio of Year 10 students to Year 11 students in a school is 2 : 4.
60% of the Year 10 students and 15% of the Year 11 students have a part-time job.
What percentage of all of these students have a part-time job?

Imagine 120 students in total: 40 Year 10 students and 80 Year 11 students
60% of 40 = 24 and 15% of 80 = 12
Together: 36 out of 120
36 ÷ 120 × 100 = 30%
Answer: 30%

4 Completing the square

(a) Write $x^2 - 10x + 18$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 - 10x + 18$

(a) Half of -10 is -5, so $x^2 - 10x = (x - 5)^2 - 25$
$x^2 - 10x + 18 = (x - 5)^2 - 25 + 18$
$= (x - 5)^2 - 7$
(b) The minimum is when $x - 5 = 0$, so $x = 5$ and $y = -7$
Answer: (a) $(x - 5)^2 - 7$ (b) $(5, -7)$

5 Recasting solids

A solid metal sphere of radius 6 cm is melted down and recast into a solid cylinder of radius 3 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 6^3 = 288\pi$

Volume of cylinder = $\pi \times 3^2 \times h = 9\pi h$
$9h = 288$
$h = 32$ cm
Answer: 32 cm

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6 Prime factors and indices

Write $6^4 \times 36^3$ as a product of powers of prime numbers.

$6 = 2 \times 3$, $36 = 2^2 \times 3^2$

$6^4 = 2^4 \times 3^4$

$36^3 = 2^6 \times 3^6$

Add the powers of each prime: $2^{10} \times 3^{10}$

Answer: $2^{10} \times 3^{10}$

7 Independent events and probability rules

A and B are two events.
P(A) = 0.8, P(B) = 0.7 and P(A or B) = 0.94
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.94 = 0.8 + 0.7 - P(A \cap B)$, so $P(A \cap B) = 0.56$
(b) $P(A) \times P(B) = 0.8 \times 0.7 = 0.56$
This equals $P(A \cap B)$, so A and B are independent
Answer: (a) 0.56 (b) Yes

8 Terms of a quadratic sequence

The nth term of a sequence is $3n^2 - 2n + 4$.
(a) Work out the 12th term.
(b) Is 183 a term of the sequence? You must show your working.

(a) $3 \times 12^2 - 2 \times 12 + 4 = 412$
(b) Solve $3n^2 - 2n + 4 = 183$, so $3n^2 - 2n - 179 = 0$
$b^2 - 4ac = (-2)^2 - 4 \times 3 \times (-179) = 2152$, which is not a square number
So n is not a whole number, and 183 is not a term
Answer: (a) 412 (b) No

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 4 cm, top radius 6 cm and base radius 18 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+4} = \frac{6}{18}$

$18h = 6(h + 4)$, so $12h = 24$ and $h = 2$

Large cone: $\frac{1}{3} \times \pi \times 18^2 \times 6 = 648\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 2 = 24\pi$

Frustum = $648\pi - 24\pi = 624\pi$ cm^3

Answer: 624π cm^3

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10 Ratio and money transferred

Jamal and Beth share some money in the ratio 2 : 1.
Jamal gives Beth £4.
The ratio of their money is now 8 : 7.
How much money do they have altogether?

Let the amounts be 2k and 1k, so the total is 3k

Jamal now has $2k - 4$, which is $\frac{8}{15}$ of the total

$15(2k - 4) = 8 \times 3k$
$30k - 60 = 24k$, so $6k = 60$ and $k = 10$
Total = 3×10 = £30
Answer: £30