Day 55 **S Tier 10-a-day: Model Solutions**

1 Three-part ratios

Uma, Carlos and Noah share some stickers.
Uma : Carlos = 5 : 6
Carlos : Noah = 2 : 4
(a) Work out the ratio Uma : Carlos : Noah.
(b) Noah gets 49 stickers more than Uma. How many stickers does Noah get?

(a) Make the Carlos part the same: 5 : 6 = 5 : 6 and 2 : 4 = 6 : 12
Uma : Carlos : Noah = 5 : 6 : 12
(b) Difference in parts = 7, so 1 part = 49 ÷ 7 = 7
Noah gets 12 × 7 = 84 stickers
Answer: (a) 5 : 6 : 12 (b) 84 stickers

2 Upper and lower bounds

a = 3 and b = 1.2,
both correct to 1 decimal place.
Work out the lower bound of a + b.

Each value is rounded, so it can be up to 0.05 above or below
$2.95 \leq a < 3.05$; $1.15 \leq b < 1.25$
Lower bound = 2.95 + 1.15 = 4.1
Answer: 4.1

3 Solving trigonometric equations

Solve $\sin x = \frac{\sqrt{3}}{2}$ for $0° \leq x \leq 360°$.

Do not use a calculator.

The related acute angle satisfies $\sin \theta = \frac{\sqrt{3}}{2}$, so $\theta = 60°$
sine is positive in the 1st and 2nd quadrants, so $x = 60°$ and $x = 180° - 60°$
$x = 60°$ and $x = 120°$
Answer: $x = 60°$ or $x = 120°$

4 Laws of indices with algebra

Simplify fully $\frac{3x^4y^3 \times 4xy^5}{4x^6y}$

Give your answer in the form kx^ay^b

Numbers: 3 × 4 ÷ 4 = 3
Powers of x: 4 + 1 − 6 = −1
Powers of y: 3 + 5 − 1 = 7
$= 3x^{-1}y^7$
Answer: $3x^{-1}y^7$

5 Reverse percentages

The price of a tablet is increased by 50%.
Later the new price is reduced by 10% in a sale.
The sale price is £324.
Work out the price before the increase. (Calculator)

Multipliers: 1.5 and 0.9
Overall multiplier = 1.5 × 0.9 = 1.35
Original × 1.35 = £324
Original = £324 ÷ 1.35 = £240
Answer: £240

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6 Area and quadratic equations

The diagram shows a rectangle of length (3x + 1) cm, width (x + 3) cm and area 171 cm^2.
(a) Show that $3x^2 + 10x - 168 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(3x + 1)(x + 3) = 171$
Expand: $3x^2 + 10x + 3 = 171$, so $3x^2 + 10x - 168 = 0$
(b) Factorise: $(x - 6)(3x + 28) = 0$
$x = 6$ ($x = -\frac{28}{3}$ is not possible as a length would be negative)
Length = 19 cm and width = 9 cm
Diagonal = $\sqrt{19^2 + 9^2} = \sqrt{442}$ = 21.0 cm (3 s.f.)
Answer: (b) 21.0 cm

7 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 7n - 4$.
(a) Work out the 10th term.
(b) Is 485 a term of the sequence? You must show your working.

(a) $2 \times 10^2 + 7 \times 10 - 4 = 266$
(b) Solve $2n^2 + 7n - 4 = 485$, so $2n^2 + 7n - 489 = 0$
$b^2 - 4ac = 7^2 - 4 \times 2 \times (-489) = 3961$, which is not a square number
So n is not a whole number, and 485 is not a term
Answer: (a) 266 (b) No

8 Inverse proportion

T is inversely proportional to the square root of n.
When n = 1, T = 15.
Find the value of T when n = 25.

$T = \frac{k}{\sqrt{n}}$
$15 = \frac{k}{1}$, so $k = 15 \times 1 = 15$
$T = \frac{15}{5} = 3$
Answer: T = 3

9 Iteration: rearranging equations

(a) Show that the equation $x^3 - 2x - 35 = 0$ has a solution between x = 3 and x = 4.
(b) Show that the equation can be rearranged to give $x = \sqrt[3]{2x + 35}$

(a) Let $f(x) = x^3 - 2x - 35$. $f(3) = 3^3 - 2 \times 3 - 35 = -14$ and $f(4) = 4^3 - 2 \times 4 - 35 = 21$
f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4
(b) Rearrange $x^3 - 2x - 35 = 0$: $x^3 = 2x + 35$
Take the cube root of both sides: $x = \sqrt[3]{2x + 35}$
Answer: Shown

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10 Surds and fractions

Simplify $\frac{2}{4+\sqrt{10}} + \frac{3}{4-\sqrt{10}}$

Give your answer with a rational denominator.

Common denominator: $(4+\sqrt{10})(4-\sqrt{10}) = 16 - 10 = 6$

Numerator: $2(4-\sqrt{10}) + 3(4+\sqrt{10})$

$= 20 + \sqrt{10}$

$\frac{20+\sqrt{10}}{6}$

Answer: $\mathbf{\frac{20+\sqrt{10}}{6}}$