Day 54 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 06:40 (London time).
It takes 9 hours 40 minutes.
When it is 12:00 in London it is 20:00 in Tokyo.
At what time, in Tokyo time, does the flight land? Say if it is on a different day.

Arrival in London time = 06:40 + 9 h 40 min = 16:20
Tokyo is 8 hours ahead of London, so add 8 hours
00:20 the next day
Answer: 00:20 the next day

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = (2x)°
Angle PQR = (3x + 5)°
Angle PRS = (4x + 34)°
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$4x + 34 = 5x + 5$
$x = 29$
(b) Angle PRS = 150° so angle PRQ = 180° − 150° = 30°
Answer: (a) x = 29 (b) 30°

3 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 10 ÷ 10 = 1 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 10 + 10 \times 30 + \frac{1}{2} \times 5 \times 10 = 50 + 300 + 25$
$= 375$
Answer: (a) 1 m/s^2 (b) 375 m

4 Area of a segment

The circle has centre O and radius 9 cm. A and B are points on the circle and angle AOB = 80°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{80}{360} \times \pi \times 9^2 = 56.549$ cm^2
Area of triangle OAB = $\frac{1}{2} \times 9 \times 9 \times \sin 80° = 39.885$ cm^2
Segment = sector − triangle = 56.549 − 39.885 = 16.664
= 16.7 cm^2 (3 s.f.)
Answer: 16.7 cm^2

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5 Probability and quadratic equations

A bag contains some sweets. 3 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{1}{5}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

P(both red) = $\frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

$\frac{6}{n(n-1)} = \frac{1}{5}$, so $n(n-1) = 30$, giving $n^2 - n - 30 = 0$

$(n-6)(n+5) = 0$, so $n = 6$ (n cannot be negative)

Number of sweets that are yellow = $6 - 3 = 3$

Answer: 3

6 Percentages of percentages

In a group of students, 30% are boys and the rest are girls.
30% of the boys and 40% of the girls walk to school.
What percentage of all the students walk to school?

Take 100 students: 30 are boys and 70 are girls

30% of 30 = 9 and 40% of 70 = 28

9 + 28 = 37 out of 100

Answer: 37%

7 Composite functions: fg and gf

$f(x) = -2x + 3$ and $g(x) = x^2 - 2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 - 2) = -2x^2 + 7$

$gf(x) = g(-2x + 3) = (-2x + 3)^2 - 2 = 4x^2 - 12x + 7$

Set them equal and rearrange: $6x^2 - 12x = 0$

Divide by 6 and factorise: $(x - 2)x = 0$

$x = 0$ or $x = 2$

Answer: x = 0 or x = 2

8 Finding the rate of interest

Hassan invested £16,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £17,504.85.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $16500 \times m^2 = 17504.85$

$m^2 = 17504.85 \div 16500 = 1.0609$

$m = \sqrt{1.0609} = 1.03$

Rate = $(1.03 - 1) \times 100 = 3\%$

Answer: 3% per year

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9 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P(−7, 24) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{24}{7}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{24}{7}) = \frac{7}{24}$

$y - 24 = \frac{7}{24}(x - (-7))$

Multiply out and rearrange: $7x - 24y = -625$
(b) On the x-axis y = 0, so $7x = -625$
$x = -\frac{625}{7}$, so T is $(-\frac{625}{7}, 0)$

Answer: (a) $7x - 24y = -625$ (b) T = $(-\frac{625}{7}, 0)$

10 Algebraic proof (harder)

Show algebraically that n(n + 1)(n + 2)(n + 3) + 1 is a square number for all positive integer values of n.

$n(n + 3) = n^2 + 3n$ and $(n + 1)(n + 2) = n^2 + 3n + 2$
Let $m = n^2 + 3n$. Then the expression is m(m + 2) + 1
$= m^2 + 2m + 1 = (m + 1)^2$
$= (n^2 + 3n + 1)^2$, which is the square of an integer
Answer: Proved