Day 53 **S Tier 10-a-day: Model Solutions**

1 Exact trigonometric values

Work out the exact value of cos 60° + sin 60°
Give your answer in its simplest form, as a fraction or surd. Do not use a calculator.

$\cos 60° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

Answer: $\mathbf{\frac{1}{2} + \frac{\sqrt{3}}{2}}$

2 Reverse percentages

The price of a tablet is increased by 30%.
Later the new price is reduced by 20% in a sale.
The sale price is £520.
Work out the price before the increase. (Calculator)

Multipliers: 1.3 and 0.8
Overall multiplier = 1.3 × 0.8 = 1.04
Original × 1.04 = £520
Original = £520 ÷ 1.04 = £500
Answer: £500

3 Perpendicular bisector

A is the point (0, 0) and B is the point (−8, 2).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−4, 1)

Gradient of AB = $-\frac{1}{4}$

Perpendicular gradient = 4
$y - 1 = 4(x - (-4))$
Rearrange to integer coefficients: $4x - y = -17$
Answer: $4x - y = -17$

4 Line and curve intersections

Solve the simultaneous equations
$y = x^2 - x - 6$
$y = -2x - 4$

Substitute: $x^2 - x - 6 = -2x - 4$
Rearrange: $x^2 + x - 2 = 0$
Factorise: $(x - 1)(x + 2) = 0$, so $x = 1$ or $x = -2$
When $x = 1$: $y = -2 \times 1 - 4 = -6$
When $x = -2$: $y = -2 \times (-2) - 4 = 0$
Answer: $x = -2$, $y = 0$ and $x = 1$, $y = -6$

5 Simple and compound interest

Priya invests £1,000 for 5 years at 5% per year simple interest.
Lena invests £1,000 for 5 years at 5% per year compound interest.
Work out how much more interest Lena earns than Priya.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 5% × 5 = £250
Compound: £1,000 × 1.05^5 = £1,276.28
Compound interest = £1,276.28 − £1,000 = £276.28
Difference = £276.28 − £250 = £26.28
Answer: £26.28

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6 Trigonometric graphs

The diagram shows the graph of $y = a \sin x + c$.
(a) Write down the value of a and the value of c.
(b) Work out the value of y when $x = 30°$.

(a) The maximum value is 4 and the minimum value is 0
$a = (4 - (0)) \div 2 = 2$
$c = (4 + (0)) \div 2 = 2$

(b) $y = 2 \sin 30° + 2$, and $\sin 30° = \frac{1}{2}$

$y = 3$

Answer: (a) a = 2, c = 2 (b) y = 3

7 Index equations

Solve $8^{3x+5} = 4^{x+4}$

Write both sides as powers of 2: $8 = 2^3$ and $4 = 2^2$
Equate the powers: $9x + 15 = 2x + 8$
$7x = -7$, so $x = -1$

Answer: x = −1

8 Estimating the median from grouped data

The table gives information about 98 masses.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 13, 25, 55, 65, 75, 98
The median is the value at position $98 \div 2 = 49$
It is in the class 30 to 40, with 25 values before it, so we need 24 more of the 30 values in this class

Median $\approx 30 + \frac{24}{30} \times 10 = 38$

Answer: 38

9 Direct and inverse proportion

T is directly proportional to the cube of l.
When l = 10, T = 6000.
(a) Find a formula for T in terms of l.
(b) Find the value of T when l = 5.

(a) $T = kl^3$

Substitute l = 10 and T = 6000: $6000 = k \times 10^3$
$k = 6$

$T = 6l^3$

(b) $T = 65^3 = 750$

Answer: (a) $T = 6l^3$ (b) 750

10 Speed and a moving train

A train travels at a constant speed of 36 km/h.
It takes 45 seconds to cross completely over a bridge which is 200 m long.
Work out the length of the train, in metres.

Speed = 36 km/h = 36,000 ÷ 3600 = 10 m/s
Distance travelled in 45 s = 10 × 45 = 450 m
This is the length of the bridge plus the length of the train
Train = 450 − 200 = 250 m

Answer: 250 m