Day 52 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

Sparkle Cleaners has a call-out fee of £4.50 plus £2.50 per hour.
Fresh Start has a call-out fee of £24 plus £1 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 20 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $4.50 + 2.50n = 24 + n$
$1.50n = 19.50$, so $n = 13$
(b) Sparkle Cleaners: £4.50 + £2.50 × 20 = £54.50
Fresh Start: £24 + £1 × 20 = £44
Fresh Start is cheaper by £10.50
Answer: (a) 13 (b) Fresh Start, by £10.50

2 Mean of a changing set

The mean of 11 numbers is 40.
One number is removed and the mean of the remaining 10 numbers is 39.2.
Work out the number that was removed.

Total of the 11 numbers = 11 × 40 = 440
Total of the 10 remaining = 10 × 39.2 = 392
Number removed = 440 − 392 = 48
Answer: 48

3 Solving trigonometric equations

Solve $3 \tan x + 4 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\tan x = -\frac{4}{3}$

Related acute angle: $\tan^{-1}(\frac{4}{3}) = 53.1°$

tangent is negative in the 2nd and 4th quadrants, so $x = 180° - 53.1°$ and $x = 360° - 53.1°$
$x = 126.9°$ and $x = 306.9°$
Answer: x = 126.9° or x = 306.9°

4 Rationalising the denominator

Rationalise the denominator of $\frac{25}{2\sqrt{2}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{2}$:
$\frac{25}{2\sqrt{2}} = \frac{25\sqrt{2}}{4}$

Answer: $\frac{25\sqrt{2}}{4}$

5 Changing the subject

Make u the subject of $v = 3\sqrt{u - 5} + 2$

Subtract 2: $v - 2 = 3\sqrt{u - 5}$
Divide by 3: $\frac{v - 2}{3} = \sqrt{u - 5}$
Square both sides: $\frac{(v - 2)^2}{9} = u - 5$
$u = \frac{(v - 2)^2}{9} + 5$
Answer: $\mathbf{u = \frac{(v - 2)^2}{9} + 5}$

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6 Rates of work

Pipe A can fill an empty pool in 12 minutes.
Pipe B can fill an empty pool in 60 minutes.
Both are turned on at the same time.
How long do they take to fill an empty pool?

In 1 minute: Pipe A does $\frac{1}{12}$ and Pipe B does $\frac{1}{60}$

Together: $\frac{1}{12} + \frac{1}{60} = \frac{72}{720}$

Time = $\frac{720}{72}$ = 10 minutes

Answer: 10 minutes

7 Tangent to a circle

A circle has equation $x^2 + y^2 = 25$.
The point P(−3, 4) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (0, 0). The gradient of the radius to P is $\frac{4-0}{-3-0} = -\frac{4}{3}$

The tangent is perpendicular to the radius, so its gradient is $\frac{3}{4}$. Using P: $y - 4 = \frac{3}{4}(x + 3)$

Multiply by 4: $4(y - 4) = 3(x + 3)$
Expand and rearrange: $-3x + 4y = 25$
Multiply by −1: $3x - 4y = -25$
Answer: $3x - 4y = -25$

8 Fractional and negative indices

Work out $(\frac{64}{27})^{-\frac{1}{3}}$

A negative power means take the reciprocal: $(\frac{64}{27})^{-\frac{1}{3}} = (\frac{27}{64})^{\frac{1}{3}}$

The denominator 3 means the cube root: the cube root of $\frac{27}{64}$ is $\frac{3}{4}$

The numerator 1 means raise to the power 1: $(\frac{3}{4})^1 = \frac{3}{4}$

Answer: $\frac{3}{4}$

9 Arc length and sector area

A sector of a circle has angle 90° and area 9π cm^2.
(a) Work out the radius of the circle.
(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{90}{360} \times \pi \times r^2$, so $\frac{90}{360} \times r^2 = 9$

$r^2 = 9 \times \frac{360}{90} = 36$

r = 6 cm

(b) Arc length = $\frac{90}{360} \times 2 \times \pi \times 6 = 3\pi$ cm

Answer: (a) 6 cm (b) 3π cm

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10 Three-part ratios

Sofia, Tariq and Hassan share some stickers.
Sofia : Tariq = 2 : 9
Tariq : Hassan = 7 : 4
(a) Work out the ratio Sofia : Tariq : Hassan.
(b) They share 226 stickers altogether. How many stickers does Tariq get?

(a) Make the Tariq part the same: 2 : 9 = 14 : 63 and 7 : 4 = 63 : 36
Sofia : Tariq : Hassan = 14 : 63 : 36
(b) Total parts = 113, so 1 part = 226 ÷ 113 = 2
Tariq gets 63 × 2 = 126 stickers
Answer: (a) 14 : 63 : 36 (b) 126 stickers