Day 50 **S Tier 10-a-day: Model Solutions**

1 Solving trigonometric equations

Solve $7\cos x + 1 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\cos x = -\frac{1}{7}$

Related acute angle: $\cos^{-1}(\frac{1}{7}) = 81.8°$

cosine is negative in the 2nd and 3rd quadrants, so $x = 180° - 81.8°$ and $x = 180° + 81.8°$
$x = 98.2°$ and $x = 261.8°$
Answer: $x = 98.2°$ or $x = 261.8°$

2 Factorising fully

Factorise fully $2x^3 - 32x$

Take out the common factor 2x: $2x(x^2 - 16)$
$x^2 - 16$ is a difference of two squares: $(x - 4)(x + 4)$
Answer: $2x(x - 4)(x + 4)$

3 Percentage change

A 5% increase followed by a 50% decrease is equivalent to a single percentage change.
Work out the single percentage change that has the same effect.

Multipliers: 1.05 and 0.5
Overall multiplier = $1.05 \times 0.5 = 0.525$
$0.525 - 1 = -0.475$, so an overall decrease of 47.5%
Answer: 47.5% decrease

4 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-\mathbf{b}$, OQ = $3\mathbf{a} - 10\mathbf{b}$ and OR = $5\mathbf{a} - 16\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) $PQ = OQ - OP = 3\mathbf{a} - 9\mathbf{b}$
$QR = OR - OQ = 2\mathbf{a} - 6\mathbf{b}$

$QR = \frac{2}{3}PQ$, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line
(b) PQ : QR = 3 : 2
Answer: (b) 3 : 2

5 Fraction problems

A bag contains marbles.
$\frac{1}{4}$ of the marbles are yellow and $\frac{1}{3}$ of the marbles are black.
The rest of the marbles, 10 of them, are white.
How many yellow marbles are there?

Fraction that are white $= 1 - \frac{1}{4} - \frac{1}{3} = \frac{5}{12}$

$\frac{5}{12}$ of the total is 10

Total $= 10 \div \frac{5}{12} = 24$

Yellow marbles $= \frac{1}{4}$ of $24 = 6$

Answer: 6

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6 Equation of a circle

A and B are the points (6, −6) and (12, −16).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{6+12}{2}, \frac{-6+(-16)}{2}) = (9, -11)$

$r^2 = (6-9)^2 + (-6-(-11))^2 = 9 + 25 = 34$

$(x-9)^2 + (y+11)^2 = 34$

Answer: $(x-9)^2 + (y+11)^2 = 34$

7 Exact lengths and areas

A rectangle has area $(-1+3\sqrt{5})$ cm^2 and length $(3+2\sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{-1+3\sqrt{5}}{3+2\sqrt{5}}$

Multiply top and bottom by $(3-2\sqrt{5})$:

Numerator: $(-1+3\sqrt{5})(3-2\sqrt{5}) = -33 + 11\sqrt{5}$

Denominator: $9 - 20 = -11$

Width = $3 - \sqrt{5}$

Answer: $3 - \sqrt{5}$ cm

8 Line of best fit

The scatter graph shows age of car (x) and value (y).
The line of best fit passes through (2, 18) and (8, 9).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{9-18}{8-2} = \frac{-9}{6} = -1.5$

$y = -1.5x + c$: using (2, 18), $18 = -1.5 \times 2 + c$, so $c = 21$

$y = -1.5x + 21$

(b) For each extra year of age, the value of the car (in £100s) goes down by 1.5

Answer: (a) $y = -1.5x + 21$ (b) For each extra year of age, the value of the car (in £100s) goes down by 1.5

9 Algebraic proof (harder)

Show algebraically that the difference between the cubes of any two consecutive integers is always an odd number.

Let the integers be n and n + 1.

$(n+1)^3 - n^3 = n^3 + 3n^2 + 3n + 1 - n^3 = 3n^2 + 3n + 1$

$= 3n(n+1) + 1$

n(n + 1) is even, so 3n(n + 1) is even, and adding 1 gives an odd number

Answer: Proved

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10 Ratio and mixtures

Container A holds juice and water in the ratio 1 : 5.
Container B holds juice and water in the ratio 1 : 1.
18 litres from container A is mixed with some of container B.
The ratio of juice to water in the mixture is 3 : 7.
Work out how many litres were taken from container B.

Let y litres be taken from B

juice: $3 + \frac{1y}{2}$

water: $15 + \frac{1y}{2}$

Ratio 3 : 7, so $7(3 + \frac{1y}{2}) = 3(15 + \frac{1y}{2})$

$(2)y = 24$, so $y = 12$

Answer: 12 litres