Day 5 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $\frac{2p - q}{r}$

when p = 4, q = −1 and r = −4.

Substitute the values: $\frac{2 \times 4 - (-1)}{-4}$

$= \frac{9}{-4}$

$= -\frac{9}{4}$

Answer: $\mathbf{-\frac{9}{4}}$

2 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 4^2 + 12^2 = 160$
$AG^2 = AC^2 + CG^2 = 160 + 3^2 = 169$
AG = 13 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{3}{\sqrt{160}}$

angle GAC = 13.3° (1 d.p.)
Answer: (a) 13 cm (b) 13.3°

3 Exact lengths and areas

A rectangle has area $(19 + 11\sqrt{2})$ cm^2 and length $(3 + \sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\frac{19 + 11\sqrt{2}}{3 + \sqrt{2}}$

Multiply top and bottom by $(3 - \sqrt{2})$:
Numerator: $(19 + 11\sqrt{2})(3 - \sqrt{2}) = 35 + 14\sqrt{2}$
Denominator: $9 - 2 = 7$
Width = $5 + 2\sqrt{2}$
Answer: $\mathbf{5 + 2\sqrt{2}}$ **cm**

4 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 3 cm and inner radius 2.5 cm. (Calculator)
The density of the metal is 7.3 g/cm^3.
The mass of the pipe is 3.78 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 3780 g, so volume = mass ÷ density = 3780 ÷ 7.3 = 517.81 cm^3
Cross-sectional area = $\pi \times 3^2 - \pi \times 2.5^2 = 8.6394$ cm^2
Length = volume ÷ area = 517.81 ÷ 8.6394 = 59.9 cm (3 s.f.)
Answer: 59.9 cm

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5 Exchange rates

Hassan changes £800 into euros at £1 = €1.15.
Hassan spends €490 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Hassan get back?

£800 × 1.15 = €920
920 − 490 = €430
430 ÷ 1.6 = £268.75
Answer: £268.75

6 Recurring decimals

Without using a calculator, work out $0.\dot{5}\dot{7} + 0.6\dot{0}\dot{2}$
Give your answer as a fraction in its simplest form.

$0.\dot{5}\dot{7} = \frac{19}{33}$

$0.6\dot{0}\dot{2} = \frac{298}{495}$

$\frac{19}{33} + \frac{298}{495} = \frac{53}{45}$

Answer: $\frac{53}{45}$

7 Trigonometric graphs

The diagram shows the graph of y = a cos bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 210°.

(a) The maximum value is 2 and the minimum is −2, so a = 2
The graph completes 3 full cycles between 0° and 360°, so b = 3
(b) y = 2 cos (3 × 210°) = 2 cos 630°
cos 270° = 0, so y = 0
Answer: (a) a = 2, b = 3 (b) y = 0

8 Iteration: rearranging equations

(a) Show that the equation $x^3 + 3x - 34 = 0$ has a solution between x = 2 and x = 3.
(b) Show that the equation can be rearranged to give $x = \sqrt[3]{34 - 3x}$

(a) Let $f(x) = x^3 + 3x - 34$. $f(2) = 2^3 + 3 \times 2 - 34 = -20$ and $f(3) = 3^3 + 3 \times 3 - 34 = 2$
f(2) is negative and f(3) is positive. There is a change of sign, so there is a solution between 2 and 3
(b) Rearrange $x^3 + 3x - 34 = 0$: $x^3 = 34 - 3x$
Take the cube root of both sides: $x = \sqrt[3]{34 - 3x}$
Answer: Shown

9 HCF and LCM

The HCF of two numbers is 4 and their LCM is 132.
One of the numbers is 44.
Work out the other number.

HCF × LCM = product of the two numbers
4 × 132 = 528
Other number = 528 ÷ 44 = 12
Answer: 12

10 Tree diagrams and reverse probability

Machine A makes 30% of the bolts in a factory. Machine B makes the rest.
60% of the bolts from machine A are faulty.
10% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.3 × 0.6 + 0.7 × 0.1 = 0.18 + 0.07 = 0.25
(b) P(A and faulty) = 0.3 × 0.6 = 0.18

P(A | faulty) = 0.18 ÷ 0.25 = $\frac{18}{25}$

Answer: (a) 0.25 (b) $\frac{18}{25}$