Day 49 **S Tier 10-a-day: Model Solutions**

1 Compound interest

Zane invests £1,500 at 6% per year compound interest.
After how many whole years will the investment first be worth more than £3,300?
(Calculator)

Multiplier = 1.06
After 13 years: $1500 \times 1.06^{13} = 3199.39$
After 14 years: $1500 \times 1.06^{14} = 3391.36$
3391.36 is the first value above £3,300
Answer: 14 years

2 Laws of indices with algebra

Simplify $(4x^6y^2)^{\frac{1}{2}}$

Raise each part to the power $\frac{1}{2}$

$4^{\frac{1}{2}} = \sqrt{4} = 2$
$(x^6)^{\frac{1}{2}} = x^3$ and $(y^2)^{\frac{1}{2}} = y^1$
$= 2x^3y$
Answer: $2x^3y$

3 Exchange rates

Noah changes £300 into euros at £1 = €1.18.
Noah spends €240 and then changes the rest back into pounds at £1 = €1.6.
How many pounds does Noah get back?

£300 × 1.18 = €354
354 − 240 = €114
114 ÷ 1.6 = £71.25
Answer: £71.25

4 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 60°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 60 = 120°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 120 = 60°
Answer: (a) 120° (b) 60°

5 Quadratic inequalities

Solve $x^2 < 6x - 5$

Rearrange so that one side is 0: $x^2 - 6x + 5 < 0$
Solve $x^2 - 6x + 5 = 0$: $(x - 1)(x - 5) = 0$, so $x = 1$ or $x = 5$
The graph is a U shape crossing the x-axis at 1 and 5; it is below the axis between the roots
$1 < x < 5$
Answer: $1 < x < 5$

6 Surds

Given that $5\sqrt{5} = \sqrt{n}$,
find the value of n.

$5\sqrt{5} = \sqrt{25} \times \sqrt{5} = \sqrt{125}$
n = 125
Answer: n = 125

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7 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 17$

$y = -x + 5$

Substitute into the first equation: $x^2 + (-x + 5)^2 = 17$

Expand: $2x^2 - 10x + 8 = 0$

Divide by 2: $x^2 - 5x + 4 = 0$

Factorise: $(x - 1)(x - 4) = 0$, so $x = 1$ or $x = 4$

$y = -x + 5$ gives $y = 4$ and $y = 1$

Answer: x = 1, y = 4 and x = 4, y = 1

8 Histograms with an unmarked scale

The histogram shows the ages of visitors to a museum.

100 of them had an age between 30 and 50 years.

(a) How many had an age between 10 and 20 years?

(b) How many visitors are there in total?

Frequency is proportional to the area of a bar

30 to 50: bar is 5 squares high and 20 wide, area 100, which represents 100

So 1 unit of area represents $100 \div 100 = 1$

(a) 10 to 20: $1 \times 10 = 10$ units of area, so $10 \times 1 = 10$

(b) Total area = 10 + 70 + 100 + 210 = 390, so total = $390 \times 1 = 390$

Answer: (a) 10 (b) 390

9 Exact lengths and areas

A rectangle has area $(2\sqrt{2})$ cm^2 and length $(2 + 2\sqrt{2})$ cm.

Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\dfrac{2\sqrt{2}}{2 + 2\sqrt{2}}$

Multiply top and bottom by $(2 - 2\sqrt{2})$:

Numerator: $(2\sqrt{2})(2 - 2\sqrt{2}) = -8 + 4\sqrt{2}$

Denominator: $4 - 8 = -4$

Width = $2 - \sqrt{2}$

Answer: $2 - \sqrt{2}$ cm

10 Loci and regions

ABCD is a rectangle with AB = 4 cm and AD = 7 cm.

A point P inside the rectangle is such that AP ≤ 2 cm and P is nearer to AB than to AD.

Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 2 cm: P is inside the quarter circle of radius 2 cm centred at A (it fits inside the rectangle)

Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB

The bisector cuts the quarter circle into two equal halves

Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 2^2 = \frac{1}{2}\pi$ cm^2

Answer: $\frac{1}{2}\pi$ cm^2