Day 48 **S Tier 10-a-day: Model Solutions**

1 Independent events and probability rules

A and B are two events.
P(A) = 0.6, P(B) = 0.2 and P(A or B) = 0.68
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.68 = 0.6 + 0.2 - P(A \cap B)$, so $P(A \cap B) = 0.12$
(b) $P(A) \times P(B) = 0.6 \times 0.2 = 0.12$
This equals $P(A \cap B)$, so A and B are independent
Answer: (a) 0.12 (b) Yes

2 Error intervals

The distance to a village is 5.1 km, truncated to 1 decimal place.
Write down the error interval for the distance to a village, using d.

Truncating cuts off the later digits without rounding.
The smallest possible value is 5.1 and the value must be less than 5.2
$5.1 \leq d < 5.2$
Answer: $5.1 \leq d < 5.2$

3 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 29 is taken. Work out how many come from Group A.
(b) A different stratified sample has 30 from Group B. Work out the size of this sample.

Total = 225 + 250 + 100 + 150 = 725

(a) Group A: $\frac{225}{725} \times 29 = 9$

(b) Fraction sampled = $30 \div 250 = \frac{3}{25}$, so sample size = $\frac{3}{25} \times 725 = 87$

Answer: (a) 9 (b) 87

4 Standard form

Work out $(2.1 \times 10^7) + (9.5 \times 10^6)$
Give your answer in standard form.

Write the first number as a multiple of 10^6: $2.1 \times 10^7 = 21 \times 10^6$
$(21 + 9.5) \times 10^6 = 30.5 \times 10^6$
$= 3.05 \times 10^7$
Answer: 3.05×10^7

5 Three-part ratios

Zane, Liam and Nadia share some money.
Zane : Liam = 7 : 4
Liam : Nadia = 7 : 8
(a) Work out the ratio Zane : Liam : Nadia.
(b) They share £1090 altogether. How much money does Zane get?

(a) Make the Liam part the same: 7 : 4 = 49 : 28 and 7 : 8 = 28 : 32
Zane : Liam : Nadia = 49 : 28 : 32
(b) Total parts = 109, so 1 part = 1090 ÷ 109 = 10
Zane gets 49 × 10 = £490
Answer: (a) 49 : 28 : 32 (b) £490

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6 Area of a segment

The circle has centre O and radius 15 cm. A and B are points on the circle and angle AOB = 140°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{140}{360} \times \pi \times 15^2 = 274.89$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 15 \times 15 \times \sin 140° = 72.314$ cm^2

Segment = sector − triangle = 274.89 − 72.314 = 202.58

= 203 cm^2 (3 s.f.)

Answer: 203 cm^2

7 Equations in disguise

Solve $x - 9\sqrt{x} + 14 = 0$

Let $u = \sqrt{x}$: $u^2 - 9u + 14 = 0$

Factorise: $(u - 7)(u - 2) = 0$, so $u = 7$ or $u = 2$

$\sqrt{x} = 7$ or $\sqrt{x} = 2$

$x = 49$ or $x = 4$

Answer: x = 4 or x = 49

8 Fractions of fractions

Ravi cycles $\frac{4}{7}$ of a route before lunch.

After lunch he cycles $\frac{2}{3}$ of the remaining distance.

The distance he cycles after lunch is 36 km.
How long is the route, in km?

The second amount is $\frac{2}{3}$ of $\frac{3}{7}$, which is $\frac{2}{7}$ of the total

$\frac{2}{7}$ of the total is 36

Total = $36 \div \frac{2}{7} = 126$

Answer: 126 km

9 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M.
(a) Work out the height VM of the pyramid.
(b) Work out the volume of the pyramid.
The diagram is not drawn accurately.

(a) $AC^2 = 8^2 + 8^2 = 128$, so $AM^2 = (AC \div 2)^2 = 32$

$VM^2 = VA^2 - AM^2 = 6^2 - 32 = 4$

VM = 2 cm

(b) Volume = $\frac{1}{3} \times 8^2 \times 2 = \frac{128}{3}$ cm^3

Answer: (a) 2 cm (b) $\frac{128}{3}$ cm^3

10 Functions that are their own inverse

$f(x) = \frac{4x - 3}{x - 4}$ where $x \neq 4$

(a) Show that $ff(x) = x$.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(x - 4)$

$ff(x) = \frac{4(4x - 3) + (-3)(x - 4)}{(4x - 3) - 4(x - 4)}$

Top $= 16x - 12 - 3x + 12 = 13x$

Bottom $= 4x - 3 - 4x + 16 = 13$

$ff(x) = \frac{13x}{13} = x$

(b) $ff(x) = x$, so f is its own inverse: $f^{-1}(x) = \frac{4x - 3}{x - 4}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{4x - 3}{x - 4}}$