**Day 47** **S Tier 10-a-day: Model Solutions**

## 1 Distance-time graphs

The graph shows Sofia's cycle ride to a café and back.
(a) Work out Sofia's speed on the way back.
(b) Work out Sofia's average speed for the whole ride, including the stop.

(a) Return: 30 km in 1.5 hours, so speed = 30 ÷ 1.5 = 20 km/h
(b) Total distance = 2 × 30 = 60 km, total time = 3 hours
Average speed = 60 ÷ 3 = 20 km/h
**Answer: (a) 20 km/h (b) 20 km/h**

## 2 Substitution

Work out the value of $x^2 - 3yz$
when $x = 1$, $y = -3$ and $z = 4$.

Substitute the values: $1^2 - 3 \times (-3) \times 4$
$= 1 - (-36)$
$= 37$
**Answer: 37**

## 3 Mean of a changing set

The mean of 5 numbers is 13.
One number is removed and the mean of the remaining 4 numbers is 5.25.
Work out the number that was removed.

Total of the 5 numbers = 5 × 13 = 65
Total of the 4 remaining = 4 × 5.25 = 21
Number removed = 65 − 21 = 44
**Answer: 44**

## 4 Rational and irrational numbers

Here are six numbers.

$\sqrt{3} \times \sqrt{5}$, $\sqrt{\frac{9}{16}}$, $(1 + \sqrt{2})^2$, $3 + \sqrt{12}$, $\frac{\sqrt{18}}{\sqrt{2}}$, $(2 + \sqrt{5})(2 - \sqrt{5})$

Write down the irrational numbers.

Simplify each number:

$\sqrt{3} \times \sqrt{5} = \sqrt{15}$; $\sqrt{\frac{9}{16}} = \frac{3}{4}$; $(1 + \sqrt{2})^2 = 3 + 2\sqrt{2}$

$3 + \sqrt{12} = 3 + 2\sqrt{3}$; $\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$; $(2 + \sqrt{5})(2 - \sqrt{5}) = 4 - 5 = -1$

**Answer: $\sqrt{3} \times \sqrt{5}$, $(1 + \sqrt{2})^2$, $3 + \sqrt{12}$**

## 5 Sine rule

The diagram shows triangle ABC. (Calculator)
Angle BAC = 48° and angle ABC is acute.
Work out the size of angle ABC, marked θ. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{\sin \theta}{AC} = \frac{\sin A}{BC}$

$\frac{\sin \theta}{10} = \frac{\sin 48°}{11}$

$\sin \theta = \frac{10 \times \sin 48°}{11} = 0.67559$

$\theta = \sin^{-1}(0.67559) = 42.5°$ (1 d.p.)
**Answer: $\theta = 42.5°$**

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## 6 Where a line meets the axes

The line L has equation $8x + 15y = 120$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $8x = 120$, so $x = 15$. A is (15, 0)
At B, $x = 0$: $15y = 120$, so $y = 8$. B is (0, 8)
(b) $AB^2 = 15^2 + 8^2 = 289$
$AB = \sqrt{289} = 17$
**Answer: (a) A(15, 0), B(0, 8) (b) 17**

## 7 Area and quadratic equations

The diagram shows a rectangle of length $(3x + 6)$ cm, width $(2x + 3)$ cm and area 135 $cm^2$.
(a) Show that $6x^2 + 21x - 117 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(3x + 6)(2x + 3) = 135$
Expand: $6x^2 + 21x + 18 = 135$, so $6x^2 + 21x - 117 = 0$
(b) Factorise: $3(x - 3)(2x + 13) = 0$
$x = 3$ ($x = -\frac{13}{2}$ is not possible as a length would be negative)
Length = 15 cm and width = 9 cm
Diagonal $= \sqrt{15^2 + 9^2} = \sqrt{306} = 17.5$ cm (3 s.f.)
**Answer: (b) 17.5 cm**

## 8 Surds and fractions

Simplify $\frac{1}{3 + \sqrt{5}} + \frac{2}{3 - \sqrt{5}}$
Give your answer with a rational denominator.

Common denominator: $(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 5 = 4$
Numerator: $(3 - \sqrt{5}) + 2(3 + \sqrt{5})$
$= 9 + \sqrt{5}$
$\frac{9 + \sqrt{5}}{4}$
**Answer:** $\mathbf{\frac{9 + \sqrt{5}}{4}}$

## 9 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 7, 17, 28, 16
Mid-points: 5, 15, 30, 50
$\Sigma fm = 7\times5 + 17\times15 + 28\times30 + 16\times50 = 1930$, $\Sigma f = 68$
Mean = 1930 ÷ 68 = 28.4
**Answer: 28.4**

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## 10 Composite functions: fg and gf

$f(x) = 4x + 3$ and $g(x) = x^2 + 2$
Solve $fg(x) = gf(x)$

$fg(x) = f(x^2 + 2) = 4x^2 + 11$
$gf(x) = g(4x + 3) = (4x + 3)^2 + 2 = 16x^2 + 24x + 11$
Set them equal and rearrange: $12x^2 + 24x = 0$
Divide by 12 and factorise: $x(x + 2) = 0$
$x = -2$ or $x = 0$
**Answer: $x = -2$ or $x = 0$**