Day 46 **S Tier 10-a-day: Model Solutions**

1 Fraction problems

A tank is $\frac{3}{10}$ full.

18 more litres of water are added and the tank is then $\frac{3}{8}$ full.

Work out the capacity of the tank.

Fraction added = $\frac{3}{8} - \frac{3}{10} = \frac{3}{40}$

$\frac{3}{40}$ of the capacity is 18 litres

Capacity = $18 \div \frac{3}{40} = 240$

Answer: 240 litres

2 Stratified sampling

The table shows the number of members in each of four groups.

(a) A stratified sample of 20 is taken. Work out how many come from Group D.

(b) A different stratified sample has 14 from Group B. Work out the size of this sample.

Total = 50 + 70 + 30 + 50 = 200

(a) Group D: $\frac{50}{200} \times 20 = 5$

(b) Fraction sampled = $14 \div 70 = \frac{1}{5}$, so sample size = $\frac{1}{5} \times 200 = 40$

Answer: (a) 5 (b) 40

3 Rational and irrational numbers

Here are six numbers.

$\sqrt{0.25}, \sqrt{2} \times \sqrt{6}, \sqrt{2} + \sqrt{8}, \frac{1}{\sqrt{3}}, \sqrt{\frac{9}{16}}, \frac{\sqrt{20}}{\sqrt{5}}$

Write down the rational numbers.

Simplify each number:

$\sqrt{0.25} = 0.5$; $\sqrt{2} \times \sqrt{6} = \sqrt{12} = 2\sqrt{3}$; $\sqrt{2} + \sqrt{8} = 3\sqrt{2}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$; $\sqrt{\frac{9}{16}} = \frac{3}{4}$; $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{4} = 2$

Answer: $\sqrt{0.25}, \sqrt{\frac{9}{16}}, \frac{\sqrt{20}}{\sqrt{5}}$

4 Similar shapes

Two shapes are mathematically similar.

The smaller has height 21 cm and area 12 cm^2.

The larger has area $\frac{64}{3}$ cm^2.

Work out the height of the larger shape.

Ratio of areas = $\frac{64}{3} : 12 = 16 : 9$

Ratio of lengths = 4 : 3

Height = $21 \times \frac{4}{3} = 28$ cm

Answer: 28 cm

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5 Iteration: rearranging equations

(a) Show that the equation $x^3 + 2x - 24 = 0$ can be rearranged to give $x = \sqrt[3]{24 - 2x}$ (Calculator)

(b) Use the iteration formula $x_{n+1} = \sqrt[3]{24 - 2x_n}$ with $x_0 = 2$ to work out x_1 and x_2. Give each answer correct to 3 decimal places.

(a) Rearrange $x^3 + 2x - 24 = 0$: $x^3 = 24 - 2x$

Take the cube root of both sides: $x = \sqrt[3]{24 - 2x}$

(b) $x_1 = \sqrt[3]{24 - 2 \times 2} = 2.71442...$

$x_2 = \sqrt[3]{24 - 2 \times x_1} = 2.64817...$

$x_1 = 2.714$ and $x_2 = 2.648$ (3 d.p.)

Answer: (b) $x_1 = 2.714$, $x_2 = 2.648$

6 Bounds in geometry

A runner covers 800 metres, correct to the nearest 10 metres, in a time of 12.1 seconds, correct to 1 decimal place. (Calculator)

Work out the upper bound of the runner's average speed in metres per second. Give your answer correct to 3 significant figures.

Distance is between 795 and 805 metres; time is between 12.05 and 12.15 seconds

Speed = distance ÷ time. For the upper bound use the greatest distance and the smallest time

805 ÷ 12.05 = 66.8 m/s (3 s.f.)

Answer: 66.8 m/s

7 Equations in disguise

Solve $x^4 - 5x^2 + 4 = 0$

Let $u = x^2$: $u^2 - 5u + 4 = 0$

Factorise: $(u - 1)(u - 4) = 0$, so $u = 1$ or $u = 4$

$x^2 = 1$ or $x^2 = 4$

$x = \pm 1$ or $x = \pm 2$

Answer: $x = -2, -1, 1, 2$

8 Laws of indices

Write $\frac{8^{n+2} \times 4^{n+1}}{4^n}$ in the form 2^{an+c}

where a and c are integers.

$8 = 2^3$, $4 = 2^2$, $4 = 2^2$

$8^{n+2} = 2^{3(n+2)}$, $4^{n+1} = 2^{2(n+1)}$, $4^n = 2^{2n}$

Exponent: $3(n + 2) + 2(n + 1) - 2n = 3n + 8$

Answer: $a = 3$, $c = 8$

9 Tree diagrams and reverse probability

Machine A makes 40% of the bolts in a factory. Machine B makes the rest.

60% of the bolts from machine A are faulty.

30% of the bolts from machine B are faulty.

(a) Work out the probability that a bolt chosen at random is faulty.

(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.4 × 0.6 + 0.6 × 0.3 = 0.24 + 0.18 = 0.42

(b) P(A and faulty) = 0.4 × 0.6 = 0.24

P(A | faulty) = 0.24 ÷ 0.42 = $\frac{4}{7}$

Answer: (a) 0.42 (b) $\frac{4}{7}$

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10 Equations with algebraic fractions

Solve $\frac{5}{x+2} - \frac{1}{x+5} = \frac{3}{2}$

Multiply every term by $(x + 2)(x + 5)$: $5(x + 5) - (x + 2) = \frac{3}{2} \times (x + 2)(x + 5)$

Multiply by 2 to clear the fraction: $10(x + 5) - 2(x + 2) = 3(x + 2)(x + 5)$

Expand and rearrange so that one side is 0: $3x^2 + 13x - 16 = 0$

Factorise: $(x - 1)(3x + 16) = 0$

$x = -\frac{16}{3}$ or $x = 1$

Answer: $x = -\frac{16}{3}$ or $x = 1$