**Day 45** **S Tier 10-a-day: Model Solutions**

## 1 Comparing costs

Zip Taxis has a fixed charge of £1 plus £5.50 per mile.
Go Cabs has a fixed charge of £10 plus £4 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 14 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $1 + 5.50n = 10 + 4n$
$1.50n = 9$, so $n = 6$
(b) Zip Taxis: £1 + £5.50 × 14 = £78
Go Cabs: £10 + £4 × 14 = £66
Go Cabs is cheaper by £12
**Answer: (a) 6 (b) Go Cabs, by £12**

## 2 Multiplying and dividing surds

Simplify fully $5\sqrt{3} \times \sqrt{24}$

$5\sqrt{3} \times \sqrt{24} = 5\sqrt{72}$
$\sqrt{72} = 6\sqrt{2}$, so $5 \times 6\sqrt{2} = 30\sqrt{2}$
**Answer: $30\sqrt{2}$**

## 3 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is $y = -0.5x + 31$.
(a) Estimate y when x = 7.
(b) Explain why an estimate of y when x = 15 may be unreliable.

(a) $y = -0.5 \times 7 + 31 = 27.5$
(b) x = 15 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
**Answer: (a) 27.5 (b) x = 15 is outside the range of the data**

## 4 Laws of indices with algebra

Simplify fully $\frac{6x^3y^6 \times 7x^3y^5}{3x^2y^2}$

Give your answer in the form $kx^ay^b$

Numbers: $6 \times 7 \div 3 = 14$
Powers of x: $3 + 3 - 2 = 4$
Powers of y: $6 + 5 - 2 = 9$
$= 14x^4y^9$
**Answer: $14x^4y^9$**

## 5 HCF and LCM

(a) Write 135 and 300 as products of prime factors.
(b) Find the HCF and the LCM of 135 and 300.
Give your answers as numbers.

(a) $135 = 3^3 \times 5$ and $300 = 2^2 \times 3 \times 5^2$
(b) HCF = $3 \times 5 = 15$
LCM = $2^2 \times 3^3 \times 5^2 = 2700$
**Answer: (a) $3^3 \times 5$; $2^2 \times 3 \times 5^2$ (b) HCF = 15, LCM = 2700**

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## 6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -5$ and $x = 4$, so $y = a(x + 5)(x - 4)$
When $x = 0$, $y = -20$: $a \times 5 \times (-4) = -20$, so $a = 1$
$y = (x + 5)(x - 4)$
$y = x^2 + x - 20$
**Answer: $y = x^2 + x - 20$**

## 7 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 4^2 + 7^2 = 65$
$AG^2 = AC^2 + CG^2 = 65 + 4^2 = 81$
$AG = 9$ cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{65}}$
angle GAC = 26.4° (1 d.p.)
**Answer: (a) 9 cm (b) 26.4°**

## 8 Averages with algebra

Here are five numbers: $x + 2$, $2x - 2$, $x + 5$, 3, 6.
The mean of the five numbers is 10.
(a) Work out the value of x.
(b) Work out the range of the five numbers.

(a) Total = $(x + 2) + (2x - 2) + (x + 5) + (3) + (6) = 10 \times 5 = 50$
$4x + 14 = 50$, so $4x = 36$ and $x = 9$
The numbers are 11, 16, 14, 3, 6
(b) Range = $16 - 3 = 13$
**Answer: (a) x = 9 (b) 13**

## 9 Surds with unknowns

Given that $(a + \sqrt{2})(b + \sqrt{2}) = 9 + 8\sqrt{2}$,
where a and b are positive integers with $a < b$, find the values of a and b.

$(a + \sqrt{2})(b + \sqrt{2}) = ab + (a + b)\sqrt{2} + 2$
Compare the $\sqrt{2}$ terms: $a + b = 8$
Compare the integer terms: $ab + 2 = 9$, so $ab = 7$
Two numbers with sum 8 and product 7 are 1 and 7
**Answer: a = 1, b = 7**

## 10 Similar shapes

Two shapes are mathematically similar.
The smaller has height 30 cm and area 36 $cm^2$.
The larger has area 12.96 $cm^2$.
Work out the height of the larger shape.

Ratio of areas = 12.96 : 36 = 9 : 25
Ratio of lengths = 3 : 5
Height = $30 \times \frac{3}{5} = 18$ cm
**Answer: 18 cm**