Day 44 **S Tier 10-a-day: Model Solutions**

1 Surds

Given that $\sqrt{12} - \sqrt{3} = \sqrt{n}$,
find the value of n.

$\sqrt{12} = 2\sqrt{3}$
$2\sqrt{3} - \sqrt{3} = \sqrt{3}$
$\sqrt{3} = \sqrt{3}$, so n = 3
Answer: n = 3

2 Pressure

A concrete slab in the shape of a cuboid weighs 930 N.
Its smallest face has area 250 cm^2. Its largest face has area 2,500 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 250 cm^2 = 0.025 m^2 and 2,500 cm^2 = 0.25 m^2
(a) Greatest pressure is on the smallest face: 930 ÷ 0.025 = 37,200 N/m^2
(b) Least pressure is on the largest face: 930 ÷ 0.25 = 3,720 N/m^2
Answer: (a) 37,200 N/m^2 (b) 3,720 N/m^2

3 Simultaneous equations

Solve the simultaneous equations
$-x - 3y = 5$
$-4x + 2y = -22$

To match the y terms: (1) × 2 gives $-2x - 6y = 10$, and (2) × 3 gives $-12x + 6y = -66$
Add the equations to eliminate y: $-14x = -56$
x = 4
Substitute x = 4 into (1): $-3y - 4 = 5$
$-3y = 9$, so $y = -3$
Answer: x = 4, y = −3

4 Relative speed

Leila and Rhys live 280 km apart.
At 10:00 they set off towards each other on the same road.
Leila drives at 80 km/h and Rhys drives at 80 km/h.
(a) At what time do they meet?
(b) How far from Leila's home do they meet?

Closing speed = 80 + 80 = 160 km/h
(a) Time = 280 ÷ 160 = 1.75 hours = 105 minutes, so they meet at 11:45
(b) Distance = 80 × 1.75 = 140 km
Answer: (a) 11:45 (b) 140 km

5 Fraction problems

A bag contains counters.
$\frac{3}{5}$ of the counters are red and $\frac{1}{4}$ of the counters are blue.
The rest of the counters, 18 of them, are green.
How many counters are in the bag?

Fraction that are green = $1 - \frac{3}{5} - \frac{1}{4} = \frac{3}{20}$

$\frac{3}{20}$ of the total is 18

Total = $18 \div \frac{3}{20} = 120$

Answer: 120

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6 Direct and inverse proportion

F is inversely proportional to d.
When d = 4, F = 2.
(a) Find a formula for F in terms of d.
(b) Find the value of d when F = 1.

(a) $F = \frac{k}{d}$

Substitute d = 4 and F = 2: $2 = \frac{k}{4}$

k = 8

$F = \frac{8}{d}$

(b) $1 = \frac{8}{d}$, so d = 8 ÷ 1 = 8

Answer: (a) $F = \frac{8}{d}$ (b) d = 8

7 Bounds in geometry

A runner covers 500 metres, correct to the nearest 50 metres, in a time of 23.6 seconds, correct to 1 decimal place. (Calculator)
Work out the upper bound of the runner's average speed in metres per second. Give your answer correct to 3 significant figures.

Distance is between 475 and 525 metres; time is between 23.55 and 23.65 seconds
Speed = distance ÷ time. For the upper bound use the greatest distance and the smallest time
525 ÷ 23.55 = 22.3 m/s (3 s.f.)
Answer: 22.3 m/s

8 Equation of a circle

A and B are the points (0, 4) and (−12, 14).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{0 + (-12)}{2}, \frac{4 + 14}{2}) = (-6, 9)$

$r^2 = (0 - (-6))^2 + (4 - 9)^2 = 36 + 25 = 61$

$(x + 6)^2 + (y - 9)^2 = 61$

Answer: $(x + 6)^2 + (y - 9)^2 = 61$

9 Loci and regions

ABCD is a rectangle with AB = 10 cm and AD = 9 cm.
A point P inside the rectangle is such that AP ≤ 8 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 8 cm: P is inside the quarter circle of radius 8 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves

Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 8^2 = 8\pi$ cm^2

Answer: 8π cm^2

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10 Ratio and money transferred

Evan and Leila share some money in the ratio 4 : 5.
Evan gives Leila £32.
The ratio of their money is now 2 : 7.
How much money do they have altogether?

Let the amounts be 4k and 5k, so the total is 9k

Evan now has $4k - 32$, which is $\frac{2}{9}$ of the total

$9(4k - 32) = 2 \times 9k$
$36k - 288 = 18k$, so $18k = 288$ and $k = 16$
Total $= 9 \times 16 = £144$
Answer: £144