Day 42 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 21:25 (London time).
It takes 7 hours 10 minutes.
When it is 12:00 in London it is 19:00 in Singapore.
At what time, in Singapore time, does the flight land? Say if it is on a different day.

Arrival in London time = 21:25 + 7 h 10 min = 04:35 the next day
Singapore is 7 hours ahead of London, so add 7 hours
11:35 the next day
Answer: 11:35 the next day

2 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -6$ and $x = 3$, so $y = a(x + 6)(x - 3)$
When $x = 0$, $y = -18$: $a \times 6 \times (-3) = -18$, so $a = 1$
$y = (x + 6)(x - 3)$
$y = x^2 + 3x - 18$
Answer: $y = x^2 + 3x - 18$

3 Tests of divisibility

The 5-digit number 6655k is a multiple of 3.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 3 if its digit sum is a multiple of 3
Digit sum = 6 + 6 + 5 + 5 + k = 22 + k
This must be a multiple of 3, so k = 2 or 5 or 8
Answer: k = 2 or 5 or 8

4 Mean of a changing set

The mean of 5 numbers is 21.
One number is removed and the mean of the remaining 4 numbers is 13.75.
Work out the number that was removed.

Total of the 5 numbers = 5 × 21 = 105
Total of the 4 remaining = 4 × 13.75 = 55
Number removed = 105 − 55 = 50
Answer: 50

5 Solving quadratics by factorising

Solve $8x^2 + 22x - 21 = 0$

Factorise: $(2x + 7)(4x - 3) = 0$
$2x + 7 = 0$ or $4x - 3 = 0$
$x = -\frac{7}{2}$ or $x = \frac{3}{4}$
Answer: $x = -\frac{7}{2}$ or $x = \frac{3}{4}$

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6 Moving averages

The table shows the monthly sales (thousands) over six periods.
The 3-point moving average centred on Mar is 22.
(a) Work out the value of x.
(b) Work out the 3-point moving average centred on Apr.

(a) $(14 + x + 16) \div 3 = 22$
$14 + x + 16 = 66$, so $x = 36$
(b) $(36 + 16 + 8) \div 3 = 60 \div 3 = 20$
Answer: (a) x = 36 (b) 20

7 Exact lengths and areas

A rectangle has area $(18 + 12\sqrt{2})$ cm^2 and length $(2 + 2\sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\frac{18 + 12\sqrt{2}}{2 + 2\sqrt{2}}$

Multiply top and bottom by $(2 - 2\sqrt{2})$:
Numerator: $(18 + 12\sqrt{2})(2 - 2\sqrt{2}) = -12 - 12\sqrt{2}$
Denominator: $4 - 8 = -4$
Width = $3 + 3\sqrt{2}$
Answer: $3 + 3\sqrt{2}$ cm

8 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 24 cm, top radius 6 cm and base radius 24 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h + 24} = \frac{6}{24}$

$24h = 6(h + 24)$, so $18h = 144$ and $h = 8$

Large cone: $\frac{1}{3} \times \pi \times 24^2 \times 32 = 6144\pi$

Small cone: $\frac{1}{3} \times \pi \times 6^2 \times 8 = 96\pi$

Frustum = $6144\pi - 96\pi = 6048\pi$ cm^3
Answer: 6048π cm^3

9 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 3 : 1.
Tin B holds red paint and white paint in the ratio 1 : 6.
4 litres from tin A and 14 litres from tin B are mixed.
Work out the ratio of red paint to white paint in the mixture.

From A: red paint = $4 \times \frac{3}{4} = 3$ litres, white paint = 1 litres

From B: red paint = $14 \times \frac{1}{7} = 2$ litres, white paint = 12 litres

red paint: $3 + 2 = 5$ and white paint: $1 + 12 = 13$
5 : 13
Answer: 5 : 13

10 Cone made from a sector

A sector of a circle has radius 6 cm and angle 180°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{180}{360} \times 2\pi \times 6 = 6\pi$ cm

This is the circumference of the base: $2\pi r = 6\pi$, so r = 3 cm

(b) Slant height = 6 cm, so height$^2 = 6^2 - 3^2 = 27$, height = $3\sqrt{3}$ cm

Volume = $\frac{1}{3} \times \pi \times 3^2 \times 3\sqrt{3} = 9\sqrt{3}\pi$ cm^3

Answer: (a) 3 cm (b) $9\sqrt{3}\pi$ cm^3