Day 41 **S Tier 10-a-day: Model Solutions**

1 Standard form

Work out $(7.3 \times 10^{-8}) \times (2.3 \times 10^{6})$
Give your answer in standard form.

$7.3 \times 2.3 = 16.79$
$10^{-8} \times 10^{6} = 10^{-2}$
$16.79 \times 10^{-2} = 1.679 \times 10^{-1}$
Answer: 1.679×10^{-1}

2 Bounds in geometry

A right-angled triangle has hypotenuse 11.9 cm and one other side 4.3 cm, both correct to 1 decimal place. (Calculator)
Work out the lower bound of the length of the third side. Give your answer correct to 3 significant figures.

The third side is $\sqrt{\text{hypotenuse}^2 - \text{side}^2}$
For the lower bound, use the lower bound of the hypotenuse and the upper bound of the other side
$\sqrt{11.85^2 - 4.35^2} = 11.0$ cm (3 s.f.)
Answer: 11.0 cm

3 Solving quadratics by factorising

Solve $12x^2 + 20 = 31x$

Rearrange so that one side is 0: $12x^2 - 31x + 20 = 0$
Factorise: $(4x - 5)(3x - 4) = 0$
$4x - 5 = 0$ or $3x - 4 = 0$
$x = \frac{5}{4}$ or $x = \frac{4}{3}$
Answer: $x = \frac{5}{4}$ or $x = \frac{4}{3}$

4 Pressure

A concrete slab in the shape of a cuboid weighs 420 N.
Its smallest face has area 250 cm^2. Its largest face has area 2,000 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the concrete slab can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 250 cm^2 = 0.025 m^2 and 2,000 cm^2 = 0.2 m^2
(a) Greatest pressure is on the smallest face: $420 \div 0.025 = 16{,}800$ N/m^2
(b) Least pressure is on the largest face: $420 \div 0.2 = 2{,}100$ N/m^2
Answer: (a) 16,800 N/m^2 (b) 2,100 N/m^2

5 Surds

Given that $\sqrt{48} - \sqrt{12} = \sqrt{n}$,
find the value of n.

$\sqrt{48} = 4\sqrt{3}$ and $\sqrt{12} = 2\sqrt{3}$
$4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3}$
$2\sqrt{3} = \sqrt{12}$, so n = 12
Answer: n = 12

Day 41 **S Tier 10-a-day: Model Solutions**

6 Forming and solving quadratic equations

A rectangle has length (3x + 6) cm and width (x + 2) cm.
The area of the rectangle is 75 cm^2.
(a) Show that $3x^2 + 12x - 63 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(3x + 6)(x + 2) = 75$, so $3x^2 + 12x + 12 = 75$
$3x^2 + 12x - 63 = 0$
(b) $3x^2 + 12x - 63 = 0$: $3(x - 3)(x + 7) = 0$
$x = 3$ ($x = -7$ is not possible as lengths are positive)
Length = 15 cm and width = 5 cm, so perimeter = $2 \times (15 + 5) = 40$ cm
Answer: (b) 40 cm

7 Conditional probability from a table

The table shows information about 119 children.
(a) A child who chooses swimming is chosen at random. Work out the probability that this child is a boy.
(b) Two of the 119 children are chosen at random. Work out the probability that both are boys.

(a) 39 children are in the Swimming column, and 20 of them are boys
Probability = $\frac{20}{39}$
(b) There are 65 boys, so P(both) = $\frac{65}{119} \times \frac{64}{118} = \frac{2080}{7021}$
Answer: (a) $\frac{20}{39}$ (b) $\frac{2080}{7021}$

8 Equation of a circle

A circle has centre C(−4, −4) and passes through the point P(3, 1).
Find an equation of the circle.

$r^2 = CP^2 = (3 - (-4))^2 + (1 - (-4))^2$
$= 49 + 25 = 74$
$(x + 4)^2 + (y + 4)^2 = 74$
Answer: $(x + 4)^2 + (y + 4)^2 = 74$

9 Bounds of roots

The area of a circle is 96 cm^2, correct to the nearest cm^2.
Work out the upper bound of the radius of the circle.
Give your answer correct to 3 significant figures. (Calculator)

$95.5 \leq$ area < 96.5
Area = πr^2, so $r = \sqrt{\frac{\text{area}}{\pi}}$
Upper bound = $\sqrt{\frac{96.5}{\pi}} = 5.54228... = 5.54$ (3 s.f.)
Answer: 5.54 cm

Day 41 **S Tier 10-a-day: Model Solutions**

10 Bearings with the sine and cosine rules

A ship leaves a port P and travels 96 km on a bearing of 320° to Q. (Calculator)
It then travels 60 km on a bearing of 060° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is $320° - 180° = 140°$
Angle PQR = 80° (the difference between the bearings 140° and 060°)
$PR^2 = 96^2 + 60^2 - 2 \times 96 \times 60 \times \cos 80° = 10816$, so PR = 104.0 km

(b) $\cos(QPR) = \frac{96^2 + PR^2 - 60^2}{2 \times 96 \times PR}$, so angle QPR = 34.6°

R is clockwise of the direction PQ, so the bearing is 320° + 35° = 355°

Answer: (a) 104.0 km (b) 355°