Day 40 **S Tier 10-a-day: Model Solutions**

1 Comparing costs

Zip Taxis has a fixed charge of £4 plus £6 per mile.
Go Cabs has a fixed charge of £14 plus £4 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 9 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when $4 + 6n = 14 + 4n$
$2n = 10$, so $n = 5$
(b) Zip Taxis: £4 + £6 × 9 = £58
Go Cabs: £14 + £4 × 9 = £50
Go Cabs is cheaper by £8
Answer: (a) 5 (b) Go Cabs, by £8

2 Weighted percentages

The ratio of boys to girls in a school is 2 : 3.
25% of the boys and 50% of the girls walk to school.
What percentage of all of these students walk to school?

Imagine 100 students in total: 40 boys and 60 girls
25% of 40 = 10 and 50% of 60 = 30
Together: 40 out of 100
40 ÷ 100 × 100 = 40%
Answer: 40%

3 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(2x + 10)°$
Angle PQR = $(2x + 30)°$
Angle PRS = $(3x + 50)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$3x + 50 = 4x + 40$
$x = 10$
(b) Angle PRS = 80° so angle PRQ = 180° − 80° = 100°
Answer: (a) x = 10 (b) 100°

4 Completing the square

(a) Write $x^2 + 8x + 21$ in the form $(x + p)^2 + q$
(b) Write down the coordinates of the turning point of the curve $y = x^2 + 8x + 21$

(a) Half of 8 is 4, so $x^2 + 8x = (x + 4)^2 - 16$
$x^2 + 8x + 21 = (x + 4)^2 - 16 + 21$
$= (x + 4)^2 + 5$
(b) The minimum is when $x + 4 = 0$, so $x = -4$ and $y = 5$
Answer: (a) $(x + 4)^2 + 5$ (b) $(-4, 5)$

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5 Similar shapes

Two cones, P and Q, are mathematically similar.
The volume of P is 72 cm^3 and the volume of Q is 3087 cm^3.
The curved surface area of P is 28 cm^2.
Work out the curved surface area of Q.

Ratio of volumes = 72 : 3087 = $2^3 : 7^3$
So the ratio of lengths is 2 : 7

Ratio of areas = $2^2 : 7^2$, so scale factor = $\frac{49}{4}$

$28 \times \frac{49}{4} = 343$ cm^2

Answer: 343 cm^2

6 Gradient of a chord

A and B are points on the curve $y = -x^2 - 2x + 1$.
The x coordinate of A is 3 and the x coordinate of B is 7.
Work out the gradient of the line AB.

At A: $y = -14$. At B: $y = -62$
A is $(3, -14)$ and B is $(7, -62)$
Gradient = $\frac{-62 - (-14)}{7 - 3} = \frac{-48}{4} = -12$

Answer: -12

7 Reverse percentage growth and decay

A car loses 8% of its value each year.
After 3 years it is worth £9,733.60.
Work out its value when it was new. (Calculator)

Yearly multiplier = 0.92
Original $\times 0.92^3$ = £9,733.60
Original = £9,733.60 $\div 0.92^3$ = £12,500

Answer: £12,500

8 Standard form problems

A library has 2.5×10^4 books.
Each book has 2.3×10^5 words.
Work out the total number of words in all the books.
Give your answer in standard form.

Total = $(2.5 \times 10^4) \times (2.3 \times 10^5)$
$2.5 \times 2.3 = 5.75$ and $10^4 \times 10^5 = 10^9$
$= 5.75 \times 10^9$

Answer: 5.75×10^9 words

9 Density of an alloy

An alloy contains 113 g of lead and 73 g of tin.
The density of lead is 11.3 g/cm^3. The density of tin is 7.3 g/cm^3.
Work out the density of the alloy.

Volume of lead = 113 $\div$ 11.3 = 10 cm^3
Volume of tin = 73 $\div$ 7.3 = 10 cm^3
Total mass = 186 g, total volume = 20 cm^3
Density = 186 $\div$ 20 = 9.3 g/cm^3

Answer: 9.3 g/cm^3

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10 Nested surds

Show that $\sqrt{3 + 2\sqrt{2}} = 1 + \sqrt{2}$

Square the right-hand side: $(1 + \sqrt{2})^2 = 1 + 2\sqrt{2} + 2$

$= 3 + 2\sqrt{2}$

So $\sqrt{3 + 2\sqrt{2}} = 1 + \sqrt{2}$

Answer: $\boldsymbol{\sqrt{3 + 2\sqrt{2}} = 1 + \sqrt{2}}$