Day 4 **S Tier 10-a-day: Model Solutions**

1 Substitution

Work out the value of $\sqrt{b^2 - 4ac}$
when a = 3, b = 1 and c = −4.

Substitute the values: $\sqrt{1^2 - 4 \times 3 \times (-4)}$
$= \sqrt{49}$
$= 7$
Answer: 7

2 Mean of a changing set

The mean of 6 numbers is 14.5.
One number is removed and the mean of the remaining 5 numbers is 15.4.
Work out the number that was removed.

Total of the 6 numbers = 6 × 14.5 = 87
Total of the 5 remaining = 5 × 15.4 = 77
Number removed = 87 − 77 = 10
Answer: 10

3 Percentage change

The price of a phone is increased by 50%.
The new price is then decreased by 40%.
Work out the overall percentage change in the price.

Multipliers: 1.5 and 0.6
Overall multiplier = 1.5 × 0.6 = 0.9
0.9 − 1 = −0.1, so an overall decrease of 10%
Answer: 10% decrease

4 Exchange rates

Oscar changes £600 into dollars at £1 = $1.3.
Oscar spends $300 and then changes the rest back into pounds at £1 = $1.6.
How many pounds does Oscar get back?

£600 × 1.3 = $780
780 − 300 = $480
480 ÷ 1.6 = £300
Answer: £300

5 Arc length and sector area

A sector of a circle has radius 13 cm and angle 135°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{135}{360}$

Arc length = $\frac{135}{360} \times 2 \times \pi \times 13 = \frac{39}{4}\pi$ cm

(b) Area = $\frac{135}{360} \times \pi \times 13^2 = \frac{507}{8}\pi$ cm^2

Answer: (a) $\frac{39}{4}\pi$ cm (b) $\frac{507}{8}\pi$ cm^2

6 Percentage change in area and volume

The edge length of a cube is increased by 10%.
Work out the percentage change in the volume of the cube.

Side length multiplier = 1.1
Volume multiplier = $1.1^3 = 1.331$
1.331 − 1 = 0.331, so a 33.1% increase
Answer: 33.1% increase

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7 Recasting solids

A solid cone has base radius 2 cm and height 27 cm.
It is melted down and recast into a solid sphere.
Work out the radius of the sphere.

The volume of a cone is $\frac{1}{3}\pi r^2 h$ and the volume of a sphere is $\frac{4}{3}\pi r^3$.

Volume of cone = $\frac{1}{3} \times \pi \times 2^2 \times 27 = 36\pi$

$\frac{4}{3}\pi R^3 = 36\pi$, so $R^3 = 36 \times \frac{3}{4} = 27$

R = 3 cm

Answer: 3 cm

8 Percentages with an unknown rate

A population is decreased by x% in one year and then decreased by a further x% the next year.
Over the two years the population decreases by 43.75%.
Work out the value of x.

Yearly multiplier = $1 - \frac{x}{100}$

Two years: $(1 - \frac{x}{100})^2 = 0.5625$

Take the square root: $1 - \frac{x}{100} = 0.75$

$\frac{x}{100} = 0.25$, so x = 25

Answer: x = 25

9 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 10 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 40 ÷ 10 = 4 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 10 \times 40 + 40 \times 10 + \frac{1}{2} \times 10 \times 40 = 200 + 400 + 200$
= 800

Answer: (a) 4 m/s^2 (b) 800 m

10 Tree diagrams and reverse probability

The probability that it rains on a day is 0.7.
If it rains, the probability that Ali is late for school is 0.7.
If it does not rain, the probability that Ali is late is 0.3.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.7 × 0.7 + 0.3 × 0.3 = 0.49 + 0.09 = 0.58
(b) P(rain and late) = 0.49

P(rain | late) = 0.49 ÷ 0.58 = $\frac{49}{58}$

Answer: (a) 0.58 (b) $\frac{49}{58}$