Day 39 **S Tier 10-a-day: Model Solutions**

1 Prime factors and indices

Write $10^2 \times 25^2$ as a product of powers of prime numbers.

$10 = 2 \times 5$, $25 = 5^2$

$10^2 = 2^2 \times 5^2$

$25^2 = 5^4$

Add the powers of each prime: $2^2 \times 5^6$

Answer: $2^2 \times 5^6$

2 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x + 20)°$, $(2x - 25)°$, $(x + 30)°$, $(x - 10)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$7x + 15 = 540$, so $7x = 525$
$x = 75$
(b) The angles are 75°, 170°, 125°, 105°, 65°
The largest is 170°
Answer: (a) x = 75 (b) 170°

3 Laws of indices with algebra

Simplify fully $\frac{3x^{\frac{1}{4}} \times 2x^{\frac{3}{4}}}{6x^{-\frac{1}{2}}}$

Numbers: $3 \times 2 \div 6 = 1$

Add the powers for × and subtract for ÷: $\frac{1}{4} + \frac{3}{4} - (-\frac{1}{2}) = \frac{3}{2}$

$= x^{\frac{3}{2}}$

Answer: $x^{\frac{3}{2}}$

4 Ratio and algebra

In an animal shelter, the ratio of the number of cats to the number of dogs is 7 : 4.
Then 7 more cats are added and 13 dogs are removed.
The ratio is now 6 : 1.
How many cats were there at the start?

Let the numbers at the start be 7k and 4k
$(7k + 7) : (4k - 13) = 6 : 1$, so $1(7k + 7) = 6(4k - 13)$
$7k + 7 = 24k - 78$
$-17k = -85$, so $k = 5$
Number of cats at the start: $7 \times 5 = 35$
Answer: 35

5 Expanding brackets with surds

Expand and simplify $(2 - 3\sqrt{5})(2 + \sqrt{5})$
Give your answer in the form $p + q\sqrt{5}$

$(2 - 3\sqrt{5})(2 + \sqrt{5}) = 4 + 2\sqrt{5} - 6\sqrt{5} - 15$
$= -11 - 4\sqrt{5}$
Answer: $-11 - 4\sqrt{5}$

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6 Joint and inverse variation

P is directly proportional to v and inversely proportional to t.
When v = 6 and t = 3, P = 12.
Work out the value of P when v = 3 and t = 6.

$P = \frac{kv}{t}$

$12 = \frac{k \times 6}{3}$, so $k = 12 \times 3 \div 6 = 6$

$P = \frac{6 \times 3}{6} = 3$

Answer: P = 3

7 Rationalising the denominator

Write $\frac{1+\sqrt{7}}{5-\sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are rational numbers.

Multiply top and bottom by $(5 + \sqrt{7})$:
Numerator: $(1 + \sqrt{7})(5 + \sqrt{7}) = 12 + 6\sqrt{7}$
Denominator: $25 - 7 = 18$
$= \frac{2+\sqrt{7}}{3}$

Answer: $\frac{2+\sqrt{7}}{3}$

8 Equations in disguise

Solve $x^4 - 58x^2 + 441 = 0$

Let $u = x^2$: $u^2 - 58u + 441 = 0$
Factorise: $(u - 49)(u - 9) = 0$, so $u = 49$ or $u = 9$
$x^2 = 49$ or $x^2 = 9$
$x = \pm 7$ or $x = \pm 3$
Answer: x = −7, −3, 3, 7

9 3D problems: pyramids

VABCD is a pyramid on a square base. V is directly above the centre M. (Calculator)
(a) Work out AC as a simplified surd.
(b) Work out the angle between VA and the base, to 1 d.p.
The diagram is not drawn accurately.

(a) $AC^2 = 16^2 + 16^2 = 512$
$AC = \sqrt{512} = 16\sqrt{2}$ cm
(b) $AM = AC \div 2 = 8\sqrt{2}$ cm. The angle is VAM.
$\tan(VAM) = \frac{VM}{AM} = \frac{14}{\sqrt{128}}$
angle VAM = 51.1° (1 d.p.)
Answer: (a) $16\sqrt{2}$ cm (b) 51.1°

10 Functions that are their own inverse

$f(x) = \frac{-3x + 6}{3x + 3}$ where $x \neq -1$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (3x + 3)

$ff(x) = \frac{(-3)(-3x + 6) + 6(3x + 3)}{3(-3x + 6) - (-3)(3x + 3)}$

Top = $9x - 18 + 18x + 18 = 27x$

Bottom = $-9x + 18 + 9x + 9 = 27$

$ff(x) = \frac{27x}{27} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-3x + 6}{3x + 3}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{-3x + 6}{3x + 3}}$