Day 38 **S Tier 10-a-day: Model Solutions**

1 Percentage change in area and volume

The edge length of a cube is decreased by 40%.
Work out the percentage change in the volume of the cube.

Side length multiplier = 0.6

Volume multiplier = $0.6^3 = 0.216$

$0.216 - 1 = -0.784$, so a 78.4% decrease

Answer: 78.4% decrease

2 Solving trigonometric equations

Solve $2\cos x - 1 = 0$ for $0° \leq x \leq 360°$. (Calculator)
Give your answers correct to 1 decimal place.

$\cos x = \frac{1}{2}$

Related acute angle: $\cos^{-1}(\frac{1}{2}) = 60.0°$

cosine is positive in the 1st and 4th quadrants, so $x = 60.0°$ and $x = 360° - 60.0°$

$x = 60.0°$ and $x = 300.0°$

Answer: $x = 60.0°$ or $x = 300.0°$

3 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 30 is taken. Work out how many come from Group B.
(b) A different stratified sample has 27 from Group C. Work out the size of this sample.

Total = 180 + 180 + 180 + 60 = 600

(a) Group B: $\frac{180}{600} \times 30 = 9$

(b) Fraction sampled = $27 \div 180 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 600 = 90$

Answer: (a) 9 (b) 90

4 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 40 ÷ 2 = 20 cm

$OM^2 = OA^2 - AM^2 = 29^2 - 20^2 = 441$

OM = 21 cm

(b) $\sin(\text{angle AOM}) = \frac{20}{29}$

Angle AOM = 43.60°, so angle AOB = 2 × 43.60 = 87.2°

Answer: (a) 21 cm (b) 87.2°

5 Time zones

A flight leaves London at 19:15 (London time).
It takes 7 hours 20 minutes.
When it is 12:00 in London it is 16:00 in Dubai.
At what time, in Dubai time, does the flight land? Say if it is on a different day.

Arrival in London time = 19:15 + 7 h 20 min = 02:35 the next day

Dubai is 4 hours ahead of London, so add 4 hours

06:35 the next day

Answer: 06:35 the next day

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6 Equations in disguise

Solve $x^4 - 41x^2 + 400 = 0$

Let $u = x^2$: $u^2 - 41u + 400 = 0$
Factorise: $(u - 25)(u - 16) = 0$, so $u = 25$ or $u = 16$
$x^2 = 25$ or $x^2 = 16$
$x = \pm 5$ or $x = \pm 4$
Answer: $x = -5, -4, 4, 5$

7 Growth and decay

A rare car is worth £1,000.
Its value increases by 12% each year.
After how many whole years will it first be worth more than £1,500? (Calculator)

Multiplier each year = 1.12
After 3 years: £1,000 × 1.12^3 = £1,404.93
After 4 years: £1,000 × 1.12^4 = £1,573.52
So the first time it is more than £1,500 is after 4 years
Answer: 4 years

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 10) and (4, 80).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 10$ and $k\,a^4 = 80$
Divide: $a^3 = \frac{80}{10} = 8$
$a = 2$
From the first point: $k \times 2 = 10$, so $k = 5$
Answer: $k = 5$, $a = 2$

9 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 60 − 30 = 30
IQR of B = 85 − 60 = 25
(b) Class B has the higher median (80 compared with 55), so on average Class B scored higher
Class B has the smaller interquartile range (25 compared with 30), so its marks are more consistent
Answer: (a) A: 30, B: 25 (b) B has the higher median, B has the smaller IQR

10 Tangency conditions

The line $y = -2x + k$ is a tangent to the curve $y = 2x^2 + x + 4$.
Find the value of k.

Equate: $2x^2 + x + 4 = -2x + k$
Rearrange: $2x^2 + 3x + 4 - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $9 - 4 \times 2 \times (4 - k) = 0$
$8k - 23 = 0$
$k = \frac{23}{8}$
Answer: $k = \frac{23}{8}$