Day 37 **S Tier 10-a-day: Model Solutions**

1 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 68°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle OAB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 68 = 136°
(b) OA = OB (radii), so triangle OAB is isosceles
Angle OAB = (180 − 136) ÷ 2 = 22°
Answer: (a) 136° (b) 22°

2 Substitution

Work out the value of $m^2 - 3nk$
when m = −2, n = 2 and k = 3.

Substitute the values: $(-2)^2 - 3 \times 2 \times 3$
$= 4 - 18$
$= -14$
Answer: −14

3 Averages with algebra

Here are five numbers: 3, x − 1, x + 1, x + 4, 3x + 4.
The mean of the five numbers is 13.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = (3) + (x − 1) + (x + 1) + (x + 4) + (3x + 4) = 13 × 5 = 65
6x + 11 = 65, so 6x = 54 and x = 9
The numbers are 3, 8, 10, 13, 31
(b) In order: 3, 8, 10, 13, 31, so the median is 10
Answer: (a) x = 9 (b) 10

4 Percentage change in area and volume

The length of a rectangle is decreased by 30%.
The width of the rectangle is decreased by 40%.
Work out the percentage change in the area of the rectangle.

Length multiplier 0.7, width multiplier 0.6
Area multiplier = 0.7 × 0.6 = 0.42
0.42 − 1 = −0.58, so a 58% decrease
Answer: 58% decrease

5 Quadratic formula and surds

Solve $3x^2 - 9x - 3 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 3, b = −9, c = −3

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4 \times 3 \times (-3)}}{2 \times 3}$$

$$x = \frac{9 \pm \sqrt{117}}{6}$$

$\sqrt{117} = 3\sqrt{13}$, so $x = \frac{9 \pm 3\sqrt{13}}{6}$

Divide the top and bottom by 3: $x = \frac{3 \pm \sqrt{13}}{2}$

Answer: $\mathbf{x = \frac{3 \pm \sqrt{13}}{2}}$

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6 Speed and a moving train

A train travels at a constant speed of 72 km/h.
It takes 25 seconds to go completely through a tunnel which is 140 m long.
Work out the length of the train, in metres.

Speed = 72 km/h = 72,000 ÷ 3600 = 20 m/s
Distance travelled in 25 s = 20 × 25 = 500 m
This is the length of the tunnel plus the length of the train
Train = 500 − 140 = 360 m
Answer: 360 m

7 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 10 cm, top radius 2 cm and base radius 6 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+10} = \frac{2}{6}$

6 h = 2(h + 10), so 4 h = 20 and h = 5

Large cone: $\frac{1}{3} \times \pi \times 6^2 \times 15 = 180\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 5 = \frac{20}{3}\pi$

Frustum = $180\pi - \frac{20}{3}\pi = \frac{520}{3}\pi$ cm^3

Answer: $\frac{520}{3}\pi$ cm^3

8 HCF and LCM

The HCF of two numbers is 3 and their LCM is 45.
One of the numbers is 9.
Work out the other number.

HCF × LCM = product of the two numbers
3 × 45 = 135
Other number = 135 ÷ 9 = 15
Answer: 15

9 Area of a segment

The circle has centre O and radius 9 cm. A and B are points on the circle and angle AOB = 100°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{100}{360} \times \pi \times 9^2 = 70.686$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 9 \times 9 \times \sin 100° = 39.885$ cm^2

Segment = sector − triangle = 70.686 − 39.885 = 30.801
= 30.8 cm^2 (3 s.f.)
Answer: 30.8 cm^2

10 Algebraic proof (harder)

Prove that the sum of the squares of any two odd numbers is 2 more than a multiple of 8.

Let the odd numbers be 2a + 1 and 2b + 1.
$(2a + 1)^2 + (2b + 1)^2 = 4a^2 + 4a + 4b^2 + 4b + 2 = 4[a(a + 1) + b(b + 1)] + 2$
a(a + 1) and b(b + 1) are each a product of consecutive integers, so each is even
So a(a + 1) + b(b + 1) is even, and 4 × an even number is a multiple of 8
Hence the sum is 2 more than a multiple of 8
Answer: Proved