Day 365 **S Tier 10-a-day: Model Solutions**

1 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 6) cm, DE = 9 cm and BC = 21 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 6} = \frac{9}{21}$
$21x = 9(2x + 6)$
$21x = 18x + 54$
$3x = 54$
$x = 18$
(b) AB = 2x + 6 = 42 cm
Answer: (a) x = 18 (b) 42 cm

2 Equations with fractions

Solve $\frac{x + 1}{3} - \frac{x - 7}{8} = 1$

Multiply every term by 24: $8(x + 1) - 3(x - 7) = 24$
Expand: $5x + 29 = 24$
$5x = -5$
$x = -1$
Answer: x = −1

3 Area and quadratic equations

The diagram shows a rectangle of length (2x + 1) cm, width (2x − 2) cm and area 28 cm^2.
(a) Show that $4x^2 - 2x - 30 = 0$.
(b) Work out the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

(a) $(2x + 1)(2x - 2) = 28$
Expand: $4x^2 - 2x - 2 = 28$, so $4x^2 - 2x - 30 = 0$
(b) Factorise: $2(x - 3)(2x + 5) = 0$
$x = 3$ ($x = -\frac{5}{2}$ is not possible as a length would be negative)
Length = 7 cm and width = 4 cm
Diagonal = $\sqrt{7^2 + 4^2} = \sqrt{65}$ = 8.06 cm (3 s.f.)
Answer: (b) 8.06 cm

4 Reverse percentage growth and decay

The population of a town grows by 15% each year.
After 3 years the population is 3,041.
Work out the population 3 years ago. (Calculator)

Yearly multiplier = 1.15
Original × 1.15^3 = 3,041
Original = 3,041 ÷ 1.15^3 = 2,000
Answer: 2,000

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5 Enlargement

Triangle T has vertices A(−2, 3), B(−3, 2), C(4, 3).
T is enlarged with scale factor 3 and centre (0, 3).
(a) Write down the coordinates of the image of A.
(b) The area of T is 10 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (−2 − 0, 3 − 3) = (−2, 0)
Multiply by 3: (−6, 0)
Image of A = centre + this vector = (−6, 3)
(b) Area scale factor = $(3)^2 = 9$
Area = 10 × 9 = 90 square units
Answer: (a) (−6, 3) (b) 90

6 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 - 9}{x^2 + 5x} \div \frac{x^2 - 7x + 12}{x}$

Factorise: $\frac{(x - 3)(x + 3)}{x(x + 5)} \div \frac{(x - 4)(x - 3)}{x}$

Turn the second fraction upside down and multiply: $\frac{(x - 3)(x + 3)}{x(x + 5)} \times \frac{x}{(x - 4)(x - 3)}$

Cancel (x − 3) and x
$= \frac{(x + 3)}{(x - 4)(x + 5)}$
Answer: $\mathbf{\frac{(x + 3)}{(x - 4)(x + 5)}}$

7 Surds with unknowns

Given that $(a + \sqrt{2})^2 = 18 + 8\sqrt{2}$,
where a is a positive integer, find the value of a.

$(a + \sqrt{2})^2 = a^2 + 2a\sqrt{2} + 2$
Compare the $\sqrt{2}$ terms: 2a = 8, so a = 4
Check the integer terms: $a^2 + 2 = 16 + 2 = 18$
Answer: a = 4

8 Simultaneous equations

Solve the simultaneous equations
3x + y = −24
−x + y = 0

To match the x terms: (2) × 3 gives −3x + 3y = 0
Add the equations to eliminate x: 4y = −24
y = −6
Substitute y = −6 into (1): 3x − 6 = −24
3x = −18, so x = −6
Answer: x = −6, y = −6

9 Combining means

The mean of the marks in a test for the boys is 76.
The mean of the marks in a test for the girls is 58.
The mean for all of the boys and girls together is 68.
Work out the ratio of the number of boys to the number of girls.

Let there be p boys and q girls
Total = 76p + 58q, and the overall mean gives 76p + 58q = 68(p + q)
8p = 10q
p : q = 10 : 8 = 5 : 4
Answer: 5 : 4

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10 Bounds and accuracy

a = 98.2 and b = 0.439, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$98.15 \leq a < 98.25$; $0.4385 \leq b < 0.4395$
Lower bound = 98.15 ÷ 0.4395 = 223.32...
Upper bound = 98.25 ÷ 0.4385 = 224.06...
Both bounds round to 220 to 2 significant figures, but not to 3 significant figures
Answer: 220 (2 s.f.)