Day 364 **S Tier 10-a-day: Model Solutions**

1 Time zones

A flight leaves London at 20:25 (London time).
It takes 7 hours 25 minutes.
When it is 12:00 in London it is 14:00 in Cairo.
At what time, in Cairo time, does the flight land? Say if it is on a different day.

Arrival in London time = 20:25 + 7 h 25 min = 03:50 the next day
Cairo is 2 hours ahead of London, so add 2 hours
05:50 the next day
Answer: 05:50 the next day

2 Changing the subject

Make r the subject of $P = 5\sqrt{r-5} + 4$

Subtract 4: $P - 4 = 5\sqrt{r-5}$

Divide by 5: $\frac{P-4}{5} = \sqrt{r-5}$

Square both sides: $\frac{(P-4)^2}{25} = r - 5$

$r = \frac{(P-4)^2}{25} + 5$

Answer: $\mathbf{r = \frac{(P-4)^2}{25} + 5}$

3 Standard form

Work out $(5.4 \times 10^7) + (9.2 \times 10^5)$
Give your answer in standard form.

Write the first number as a multiple of 10^5: $5.4 \times 10^7 = 540 \times 10^5$
$(540 + 9.2) \times 10^5 = 549.2 \times 10^5$
$= 5.492 \times 10^7$
Answer: $\mathbf{5.492 \times 10^7}$

4 Direct proportion formulae

E is directly proportional to v.
When v = 4, E = 2.
(a) Find a formula for E in terms of v.
(b) Work out the positive value of v when E = 3.

(a) E = kv
2 = k × 4, so k = 2 ÷ 4 = 0.5
E = 0.5v
(b) 3 = 0.5v, so v = 3 ÷ 0.5 = 6
Answer: (a) E = 0.5v (b) v = 6

5 Simplifying surds

Write $\sqrt{28} + \sqrt{252} + \sqrt{175}$ in the form $a\sqrt{7}$
where a is an integer.

$\sqrt{28} = 2\sqrt{7}$; $\sqrt{252} = 6\sqrt{7}$; $\sqrt{175} = 5\sqrt{7}$
$(2 + 6 + 5)\sqrt{7} = 13\sqrt{7}$
Answer: $\mathbf{13\sqrt{7}}$

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6 Sample space and conditional probability

A spinner numbered 1 to 4 and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.
(a) Work out the probability that the sum is greater than 7.
(b) Given that the first number is even, work out the probability that the sum is greater than 7.

(a) There are 4 × 5 = 20 equally likely outcomes, and 3 of them satisfy: the sum is greater than 7

Probability = $\frac{3}{20}$

(b) 10 outcomes satisfy: the first number is even. Of these, 2 also satisfy: the sum is greater than 7

Probability = $\frac{1}{5}$

Answer: (a) $\frac{3}{20}$ (b) $\frac{1}{5}$

7 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 84 km on a bearing of 205° to Q. (Calculator)
It then travels 114 km on a bearing of 330° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 205° − 180° = 025°
Angle PQR = 55° (the difference between the bearings 025° and 330°)
$PR^2 = 84^2 + 114^2 - 2 \times 84 \times 114 \times \cos 55° = 9066.9$, so PR = 95.2 km

(b) $\cos(QPR) = \frac{84^2 + PR^2 - 114^2}{2 \times 84 \times PR}$, so angle QPR = 78.7°

R is clockwise of the direction PQ, so the bearing is 205° + 79° = 284°
Answer: (a) 95.2 km (b) 284°

8 Direct and inverse proportion

V is directly proportional to the cube of r.
When r = 2, V = 16.
(a) Find a formula for V in terms of r.
(b) Find the value of r when V = 432.

(a) $V = kr^3$

Substitute r = 2 and V = 16: $16 = k \times 2^3$

k = 2

$V = 2r^3$

(b) $432 = 2r^3$, so $r^3 = 216$, so r = 6

Answer: (a) $V = 2r^3$ (b) r = 6

9 Exponential growth and decay

A radioactive sample has a mass of 800 g. Its mass halves every 12 hours.
(a) Work out the mass after 24 hours.
(b) After how many hours will the mass first be less than 20 g?

(a) 24 hours is 2 half-lives: $800 \div 2^2 = 200$ g
(b) Halve repeatedly: 800, 400, 200, 100, 50, 25, 12.5
12.5 is the first value below 20, after 6 half-lives, so 6 × 12 = 72 hours
Answer: (a) 200 g (b) 72 hours

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 2 : 1. N is on OB with ON : NB = 1 : 1. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{2}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{2}{3}\mathbf{a} = \frac{2}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{2}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{1}{2}t\mathbf{b}$

Equate: $\frac{2}{3}s = 1 - t$ and $1 - s = \frac{1}{2}t$

Solving: $s = \frac{3}{4}$ and $t = \frac{1}{2}$

OX = $\frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}$

Answer: (a) $\frac{2}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{2}\mathbf{a} + \frac{1}{4}\mathbf{b}$