Day 363 **S Tier 10-a-day: Model Solutions**

1 Changing the subject

Make l the subject of $T = 6l^3 + 8$

Subtract 8: $T - 8 = 6l^3$

Divide by 6: $l^3 = \frac{T - 8}{6}$

Cube root: $l = \sqrt[3]{\frac{T - 8}{6}}$

Answer: $\mathbf{l = \sqrt[3]{\frac{T - 8}{6}}}$

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.57, 0.6, $\frac{1}{5}$, 52%

Convert each to a decimal: 0.57 = 0.57, 0.6 = 0.6, $\frac{1}{5}$ = 0.2, 52% = 0.52

In order: $\frac{1}{5}$, 52%, 0.57, 0.6

Answer: $\mathbf{\frac{1}{5}}$**, 52%, 0.57, 0.6**

3 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (4x − 20)°, angle BCD = (2x + 38)° and angle ABC = (3x)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(4x − 20) + (2x + 38) = 180
6x + 18 = 180, so 6x = 162
x = 27
(b) Angle ABC = 3 × 27 + 0 = 81°
Angle ADC = 180 − 81 = 99°
Answer: (a) x = 27 (b) 99°

4 Speed-time graphs

The graph shows the speed of a train during 15 seconds.
(a) Work out the acceleration during the first 6 seconds.
(b) Work out the average speed over the whole 15 seconds.

(a) Acceleration = change in speed ÷ time = (24 − 0) ÷ 6 = 4 m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 24}{2} \times 6 = 72$; rectangle: $24 \times 5 = 120$; triangle: $\frac{1}{2} \times 4 \times 24 = 48$
Total = 240 m, so average speed = 240 ÷ 15 = 16 m/s
Answer: (a) 4 m/s^2 (b) 16 m/s

5 Transformations of graphs

The point P(12, −4) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of

(a) $y = \frac{1}{2}f(x)$ (b) y = f(x) − 2

(a) $y = \frac{1}{2}f(x)$ is a vertical stretch with scale factor $\frac{1}{2}$, so the y coordinate is halved: (12, −2)
(b) y = f(x) − 2 is a translation of 2 down: (12, −6)
Answer: (a) (12, −2) (b) (12, −6)

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6 Percentages

P is 40% less than A.
A is 30% less than D.
Work out P as a percentage of D.

P = 0.6 × A and A = 0.7 × D
P = 0.6 × 0.7 × D = 0.42 × D
0.42 × 100 = 42%
Answer: 42%

7 Bounds in geometry

A right-angled triangle has hypotenuse 15.0 cm and one other side 7.3 cm, both correct to 1 decimal place. (Calculator)
Work out the lower bound of the length of the third side. Give your answer correct to 3 significant figures.

The third side is $\sqrt{\text{hypotenuse}^2 - \text{side}^2}$
For the lower bound, use the lower bound of the hypotenuse and the upper bound of the other side
$\sqrt{14.95^2 - 7.35^2}$ = 13.0 cm (3 s.f.)
Answer: 13.0 cm

8 Density of an alloy

An alloy is made by mixing 60 cm^3 of nickel with 100 cm^3 of lead.
The density of nickel is 8.9 g/cm^3. The density of lead is 11.3 g/cm^3.
Work out the density of the alloy.

Mass of nickel = 60 × 8.9 = 534 g
Mass of lead = 100 × 11.3 = 1130 g
Total mass = 1664 g, total volume = 160 cm^3
Density = 1664 ÷ 160 = 10.4 g/cm^3
Answer: 10.4 g/cm^3

9 Counting factors

N = $2^5 \times 3^2$
How many of the factors of N are odd numbers?

An odd factor contains no 2s
Ignore the 2 and count the factors of the rest: 3 = 3
Answer: 3

10 Speed and a moving train

A train travels at a constant speed of 108 km/h.
It takes 45 seconds to go completely through a tunnel which is 1,060 m long.
Work out the length of the train, in metres.

Speed = 108 km/h = 108,000 ÷ 3600 = 30 m/s
Distance travelled in 45 s = 30 × 45 = 1,350 m
This is the length of the tunnel plus the length of the train
Train = 1,350 − 1,060 = 290 m
Answer: 290 m