Day 362 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (5x + 5)°
Angle ABC = (3x − 39)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(5x + 5) + 2(3x − 39) = 180
11x − 73 = 180, so 11x = 253
x = 23
(b) Angle BAC = 5 × 23 + 5 = 120°
Answer: (a) x = 23 (b) 120°

2 Mean of a changing set

The mean of 9 numbers is 197.
A new number is added and the mean of all 10 numbers is 177.7.
Work out the new number.

Total of the 9 numbers = 9 × 197 = 1773
New total = 10 × 177.7 = 1777
New number = 1777 − 1773 = 4
Answer: 4

3 Fractional and negative indices

Work out $(\frac{125}{64})^{-\frac{2}{3}}$

A negative power means take the reciprocal: $(\frac{125}{64})^{-\frac{2}{3}} = (\frac{64}{125})^{\frac{2}{3}}$

The denominator 3 means the cube root: the cube root of $\frac{64}{125}$ is $\frac{4}{5}$

The numerator 2 means raise to the power 2: $(\frac{4}{5})^2 = \frac{16}{25}$

Answer: $\mathbf{\frac{16}{25}}$

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (2x + 5)°, angle BCD = (5x + 7)° and angle ABC = (3x + 10)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(2x + 5) + (5x + 7) = 180
7x + 12 = 180, so 7x = 168
x = 24
(b) Angle ABC = 3 × 24 + 10 = 82°
Angle ADC = 180 − 82 = 98°
Answer: (a) x = 24 (b) 98°

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5 Iteration

Use the iteration formula $x_{n+1} = 2 + \frac{9}{x_n}$ with $x_0 = 2$ (Calculator)

to work out x_1, x_2 and x_3.

Give each answer correct to 3 decimal places.

$x_1 = 2 + \frac{9}{2} = 6.50000...$

$x_2 = 2 + \frac{9}{x_1} = 3.38462...$

$x_3 = 2 + \frac{9}{x_2} = 4.65909...$

Rounded: $x_1 = 6.500$, $x_2 = 3.385$, $x_3 = 4.659$

Answer: $x_1 = 6.500$, $x_2 = 3.385$, $x_3 = 4.659$

6 Direct proportion formulae

E is directly proportional to v.

When v = 1, E = 8.

(a) Find a formula for E in terms of v.

(b) Work out the value of E when v = 5.

(a) $E = kv$

$8 = k \times 1$, so $k = 8 \div 1 = 8$

$E = 8v$

(b) $E = 8 \times 5 = 40$

Answer: (a) $E = 8v$ (b) 40

7 Exact lengths and areas

A rectangle has area $(16 + 7\sqrt{5})$ cm^2 and length $(3 + 2\sqrt{5})$ cm.

Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{16 + 7\sqrt{5}}{3 + 2\sqrt{5}}$

Multiply top and bottom by $(3 - 2\sqrt{5})$:

Numerator: $(16 + 7\sqrt{5})(3 - 2\sqrt{5}) = -22 - 11\sqrt{5}$

Denominator: $9 - 20 = -11$

Width = $2 + \sqrt{5}$

Answer: $2 + \sqrt{5}$ cm

8 Venn diagrams with algebra

The Venn diagram shows how many of 34 students study French (F) or Spanish (S).

(a) Work out the value of x.

(b) A student is chosen at random from those who study Spanish. Work out the probability that this student also studies French.

(a) Total: $(x + 2) + x + (2x - 2) + 6 = 34$

$4x + 6 = 34$, so $x = 7$

(b) Students in the second circle: $x + 2x - 2 = 7 + 12 = 19$, of whom 7 are in both

Probability = $\frac{7}{19}$

Answer: (a) $x = 7$ (b) $\frac{7}{19}$

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9 Changing the subject

Make l the subject of $T = \frac{5}{\sqrt{l}}$

Multiply by $\sqrt{l}$: $T\sqrt{l} = 5$

Divide by T: $\sqrt{l} = \frac{5}{T}$

Square both sides: $l = \frac{25}{T^2}$

Answer: $l = \frac{25}{T^2}$

10 Expanding brackets with surds

Expand and simplify $(1 - 2\sqrt{3})^2$

Give your answer in the form $p + q\sqrt{3}$

$(1 - 2\sqrt{3})^2 = 1 - 4\sqrt{3} + 12$

$= 13 - 4\sqrt{3}$

Answer: $13 - 4\sqrt{3}$