Day 361 **S Tier 10-a-day: Model Solutions**

1 Fraction problems

A bag contains balloons.

$\frac{1}{5}$ of the balloons are pink and $\frac{3}{8}$ of the balloons are silver.

The rest of the balloons, 34 of them, are gold.

How many balloons are in the bag?

Fraction that are gold = $1 - \frac{1}{5} - \frac{3}{8} = \frac{17}{40}$

$\frac{17}{40}$ of the total is 34

Total = $34 \div \frac{17}{40} = 80$

Answer: 80

2 Gradient of a chord

A and B are points on the curve $y = -x^2 + 2x + 3$.

The x coordinate of A is 3 and the x coordinate of B is 4.

Work out the gradient of the line AB.

At A: $y = 0$. At B: $y = -5$

A is (3, 0) and B is (4, −5)

Gradient = $\frac{-5 - 0}{4 - 3} = \frac{-5}{1} = -5$

Answer: −5

3 Compound interest

Jess invests £20,000.

The account pays compound interest of 3% a year for the first 2 years,

then 2% a year for the next 2 years.

Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.03 for 2 years, then 1.02 for 2 years

£20,000 × 1.03^2 × 1.02^2

= £22,075.21

Answer: £22,075.21

4 Pressure

A metal crate in the shape of a cuboid weighs 1110 N.

Its smallest face has area 800 cm^2. Its largest face has area 1,000 cm^2.

Pressure = force ÷ area

Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 800 cm^2 = 0.08 m^2 and 1,000 cm^2 = 0.1 m^2

(a) Greatest pressure is on the smallest face: $1110 \div 0.08 = 13{,}875$ N/m^2

(b) Least pressure is on the largest face: $1110 \div 0.1 = 11{,}100$ N/m^2

Answer: (a) 13,875 N/m^2 (b) 11,100 N/m^2

5 Perpendicular bisector

A is the point (−3, 2) and B is the point (−1, −2).

Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (−2, 0)

Gradient of AB = −2

Perpendicular gradient = $\frac{1}{2}$

$y - 0 = \frac{1}{2}(x - (-2))$

Rearrange to integer coefficients: $x - 2y = -2$

Answer: $x - 2y = -2$

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6 Reverse percentages

The price of a sofa is increased by 20%.
The new price is then increased by 8%.
The final price is £259.20.
Work out the original price of the sofa.

Multipliers: 1.2, 1.08
Overall multiplier = 1.2 × 1.08 = 1.296
Original = final ÷ multiplier = £259.20 ÷ 1.296
= £200
Answer: £200

7 Rearranging when the subject appears twice

Make x the subject of $y = \frac{6x + 4}{x - 2}$

Multiply by $x - 2$: $y(x - 2) = 6x + 4$
Expand: $xy - 2y = 6x + 4$
Collect the x terms: $xy - 6x = 2y + 4$
Factorise: $x(y - 6) = 2y + 4$
$x = \frac{2y + 4}{y - 6}$
Answer: $\mathbf{x = \frac{2y + 4}{y - 6}}$

8 Combined transformations

The point P(1, −5) is reflected in the line y = x.
The image is then reflected in the x-axis.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−5, 1)
After the second transformation: P'' = (−5, −1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (y, x) → (y, −x)
The single mapping is (x, y) → (y, −x)
This is a rotation 90° clockwise about the origin
Answer: (a) (−5, −1) (b) Rotation 90° clockwise about the origin

9 Functions that are their own inverse

$f(x) = \frac{4x + 2}{x - 4}$ where $x \neq 4$

(a) Show that ff(x) = x.

(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by $(x - 4)$
$ff(x) = \frac{4(4x + 2) + 2(x - 4)}{(4x + 2) - 4(x - 4)}$
Top = 16x + 8 + 2x − 8 = 18x
Bottom = 4x + 2 − 4x + 16 = 18
$ff(x) = \frac{18x}{18} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{4x + 2}{x - 4}$

Answer: (b) $\mathbf{f^{-1}(x) = \frac{4x + 2}{x - 4}}$

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10 Percentage change and proportion

The time taken for a journey is inversely proportional to the speed.
The speed is increased by 150%.
Work out the percentage change in the time taken for a journey.

The speed is multiplied by 2.5
The multiplier for the time taken for a journey is 1 ÷ 2.5 = 0.4
0.4 is a decrease of 60%
Answer: 60% decrease