Day 360 **S Tier 10-a-day: Model Solutions**

1 Ordering numbers

Write these numbers in order of size, starting with the smallest.

79%, $\frac{17}{20}$, 42%, 38%

Convert each to a decimal: 79% = 0.79, $\frac{17}{20}$ = 0.85, 42% = 0.42, 38% = 0.38

In order: 38%, 42%, 79%, $\frac{17}{20}$

Answer: 38%, 42%, 79%, $\frac{17}{20}$

2 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x + 15)°$, $(2x - 20)°$, $(x + 30)°$, $(x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$7x + 50 = 540$, so $7x = 490$
$x = 70$
(b) The angles are 70°, 155°, 120°, 100°, 95°
The largest is 155°
Answer: (a) x = 70 (b) 155°

3 Changing the subject

Make l the subject of $T = 7l^2 - 11$
where l is positive.

Add 11: $T + 11 = 7l^2$

Divide by 7: $l^2 = \frac{T + 11}{7}$

Square root (positive): $l = \sqrt{\frac{T + 11}{7}}$

Answer: $l = \sqrt{\frac{T + 11}{7}}$

4 Ratio and mixtures

Tin A holds red paint and white paint in the ratio 3 : 1.
Tin B holds red paint and white paint in the ratio 5 : 1.
12 litres from tin A is mixed with some of tin B.
The ratio of red paint to white paint in the mixture is 19 : 5.
Work out how many litres were taken from tin B.

Let y litres be taken from B

red paint: $9 + \frac{5y}{6}$

white paint: $3 + \frac{1y}{6}$

Ratio 19 : 5, so $5(9 + \frac{5y}{6}) = 19(3 + \frac{1y}{6})$

$(1)y = 12$, so $y = 12$
Answer: 12 litres

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5 Percentages

A is 10% more than B.
B is 15% more than R.
Work out A as a percentage of R.

A = 1.1 × B and B = 1.15 × R
A = 1.1 × 1.15 × R = 1.265 × R
1.265 × 100 = 126.5%
Answer: 126.5%

6 Bounds in geometry

A right-angled triangle has hypotenuse 16.6 cm and one other side 7.6 cm, both correct to 1 decimal place. (Calculator)
Work out the upper bound of the length of the third side. Give your answer correct to 3 significant figures.

The third side is $\sqrt{\text{hypotenuse}^2 - \text{side}^2}$
For the upper bound, use the upper bound of the hypotenuse and the lower bound of the other side
$\sqrt{16.65^2 - 7.55^2}$ = 14.8 cm (3 s.f.)
Answer: 14.8 cm

7 Profit and loss

A retailer buys a games console and adds a 30% mark-up to get the selling price.
In a sale the selling price is reduced by 20% to £72.80.
Work out how much the shop paid for a games console.

Overall multiplier = 1.3 × 0.8 = 1.04
Cost = £72.80 ÷ 1.04
= £70
Answer: £70

8 Tangency conditions

The line $y = -x + k$ is a tangent to the curve $y = x^2 + 5x$.
Find the value of k.

Equate: $x^2 + 5x = -x + k$
Rearrange: $x^2 + 6x - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $36 - 4 \times 1 \times (-k) = 0$
4k + 36 = 0
k = −9
Answer: k = −9

9 Estimating the mean from a histogram table

The table gives information about the ages of some people.
Work out an estimate for the mean age.
Give your answer correct to 1 decimal place. (Calculator)

Frequency = frequency density × class width: 7, 4, 21, 36
Mid-points: 5, 20, 45, 80
Σfm = 7×5 + 4×20 + 21×45 + 36×80 = 3940, Σf = 68
Mean = 3940 ÷ 68 = 57.9
Answer: 57.9

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10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 2. N is on OB with ON : NB = 3 : 2. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{3}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{3}\mathbf{a} = \frac{1}{3}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{3}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{3}{5}t\mathbf{b}$

Equate: $\frac{1}{3}s = 1 - t$ and $1 - s = \frac{3}{5}t$

Solving: $s = \frac{1}{2}$ and $t = \frac{5}{6}$

OX = $\frac{1}{6}\mathbf{a} + \frac{1}{2}\mathbf{b}$

Answer: (a) $\frac{1}{3}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{1}{6}\mathbf{a} + \frac{1}{2}\mathbf{b}$