Day 36 **S Tier 10-a-day: Model Solutions**

1 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £25,000 is taxed at 10%.
All income above that is taxed at 30%.
Gareth pays £12,850 in income tax in a year.
Work out Gareth's income for the year.

Tax on the 10% band = 10% of £25,000 = £2,500
Tax paid at 30% = £12,850 − £2,500 = £10,350
Income taxed at 30% = £10,350 ÷ 0.3 = £34,500
Income = £15,000 + £25,000 + £34,500 = £74,500
Answer: £74,500

2 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 6^2 + 17^2 = 325$
$AG^2 = AC^2 + CG^2 = 325 + 6^2 = 361$
AG = 19 cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{6}{\sqrt{325}}$

angle GAC = 18.4° (1 d.p.)
Answer: (a) 19 cm (b) 18.4°

3 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x + 1) cm, (x + 8) cm and (x + 10) cm.
The longest side is (x + 10) cm.
(a) Show that $x^2 - 2x - 35 = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x + 1)^2 + (x + 8)^2 = (x + 10)^2$
Expand: $x^2 + 2x + 1 + x^2 + 16x + 64 = x^2 + 20x + 100$
Rearrange: $x^2 - 2x - 35 = 0$
(b) Factorise: $(x - 7)(x + 5) = 0$, so x = 7 (the other root, x = −5, does not give valid side lengths)
The shorter sides are 8 cm and 15 cm, so area = $\frac{1}{2} \times 8 \times 15 = 60$ cm^2

Answer: (b) 60 cm^2

4 Rationalising the denominator

Rationalise the denominator of $\frac{8}{3\sqrt{5}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{5}$:
$\frac{8}{3\sqrt{5}} = \frac{8\sqrt{5}}{15}$

Answer: $\frac{8\sqrt{5}}{15}$

Day 36 **S Tier 10-a-day: Model Solutions**

5 Algebraic proof

Prove algebraically that the sum of the squares of any three consecutive integers is always 2 more than a multiple of 3.

Let the integers be $n - 1$, n and $n + 1$.

$(n - 1)^2 + n^2 + (n + 1)^2 = n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$

$= 3n^2 + 2$

$3n^2$ is a multiple of 3, so the sum is 2 more than a multiple of 3

Answer: Proved

6 Venn diagrams with algebra

The Venn diagram shows how many of 25 students have a tablet (T) or a laptop (L).

(a) Work out the value of x.

(b) A student is chosen at random from those who have a laptop. Work out the probability that this student also has a tablet.

(a) Total: $(x + 4) + x + (2x - 1) + 6 = 25$

$4x + 9 = 25$, so $x = 4$

(b) Students in the second circle: $x + 2x - 1 = 4 + 7 = 11$, of whom 4 are in both

Probability = $\frac{4}{11}$

Answer: (a) x = 4 (b) $\frac{4}{11}$

7 Rearranging when the subject appears twice

Make u the subject of $v = \frac{3u + 3}{u - 5}$

Multiply by $u - 5$: $v(u - 5) = 3u + 3$

Expand: $uv - 5v = 3u + 3$

Collect the u terms: $uv - 3u = 5v + 3$

Factorise: $u(v - 3) = 5v + 3$

$u = \frac{5v + 3}{v - 3}$

Answer: $\mathbf{u = \frac{5v + 3}{v - 3}}$

8 Percentages of percentages

In a group of visitors, 25% are adults and the rest are children.

30% of the adults and 45% of the children buy a souvenir.

What percentage of all the visitors buy a souvenir?

Take 100 visitors: 25 are adults and 75 are children

30% of 25 = 7.5 and 45% of 75 = 33.75

7.5 + 33.75 = 41.25 out of 100

Answer: 41.25%

9 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 80 students to complete a puzzle.

(a) Find an estimate for the median and the interquartile range.

(b) Estimate how many of the 80 had a time greater than 15.

(a) Median at cumulative frequency 40, so median = 30

Lower quartile at 20, so LQ = 20. Upper quartile at 60, so UQ = 40

Interquartile range = 40 − 20 = 20

(b) At 15, cumulative frequency = 12

Number greater = 80 − 12 = 68

Answer: (a) median 30, IQR 20 (b) 68

Day 36 **S Tier 10-a-day: Model Solutions**

10 Transformations of graphs

The graph of y = f(x) has a maximum point at (−2, 3).
Write down the coordinates of the turning point of the graph of
(a) y = f(x) − 2
(b) y = f(x − 4) + 2

(a) Start with the turning point (−2, 3)
Apply the transformation to the point: (−2, 1)
(b) f(x − 4) changes the x-coordinate to 2, and f(x) + 2 changes the y-coordinate to 5
Turning point: (2, 5)
Answer: (a) (−2, 1) (b) (2, 5)