Day 359 **S Tier 10-a-day: Model Solutions**

1 Inequalities

(a) Solve $-3 - 2x < 11$
(b) n is an integer. Write down all the values of n that satisfy
$-19 < 4n - 3 < 1$

(a) Add 3: $-2x < 14$
Divide by -2 and reverse the inequality sign: $x > -7$
(b) Add 3: $-16 < 4n < 4$
Divide by 4: $-4 < n < 1$
Integers: $-3, -2, -1, 0$
Answer: (a) $x > -7$ (b) $-3, -2, -1, 0$

2 Conditional probability from a table

The table shows information about 110 employees.
(a) An employee who travels to work by car is chosen at random. Work out the probability that this employee is a woman.
(b) Two of the 110 employees are chosen at random. Work out the probability that both are women.

(a) 32 employees are in the Car column, and 18 of them are women

Probability = $\frac{9}{16}$

(b) There are 66 women, so P(both) = $\frac{66}{110} \times \frac{65}{109} = \frac{39}{109}$

Answer: (a) $\frac{9}{16}$ (b) $\frac{39}{109}$

3 Laws of indices with algebra

Simplify $(8x^6y^{-3})^{\frac{1}{3}}$

Raise each part to the power $\frac{1}{3}$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
$(x^6)^{\frac{1}{3}} = x^2$ and $(y^{-3})^{\frac{1}{3}} = y^{-1}$
$= 2x^2y^{-1}$
Answer: $2x^2y^{-1}$

4 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 40°$
$x^2 = 73.663$
$x = 8.6$ cm (1 d.p.)
(b) Area = $\frac{1}{2} \times 8 \times 13 \times \sin 40° = 33.4$ cm^2 (3 s.f.)

Answer: (a) 8.6 cm (b) 33.4 cm^2

5 Standard form

Work out $(2.5 \times 10^5) - (5.3 \times 10^2)$
Give your answer in standard form.

Write the first number as a multiple of 10^2: $2.5 \times 10^5 = 2500 \times 10^2$
$(2500 - 5.3) \times 10^2 = 2494.7 \times 10^2$
$= 2.4947 \times 10^5$
Answer: 2.4947×10^5

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6 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve. It crosses the axes at the points shown.
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The roots are $x = -3$ and $x = 2$, so $y = a(x + 3)(x - 2)$
When $x = 0$, $y = -12$: $a \times 3 \times (-2) = -12$, so $a = 2$
$y = 2(x + 3)(x - 2)$
$y = 2x^2 + 2x - 12$
Answer: $y = 2x^2 + 2x - 12$

7 Percentage change in area and volume

The edge length of a cube is decreased by 10%.
Work out the percentage change in the volume of the cube.

Side length multiplier = 0.9
Volume multiplier = $0.9^3 = 0.729$
$0.729 - 1 = -0.271$, so a 27.1% decrease
Answer: 27.1% decrease

8 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 4^2 + 3^2 = 25$
$AG^2 = AC^2 + CG^2 = 25 + 12^2 = 169$
AG = 13 cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{12}{\sqrt{25}}$
angle GAC = 67.4° (1 d.p.)
Answer: (a) 13 cm (b) 67.4°

9 Tree diagrams and reverse probability

Machine A makes 40% of the bolts in a factory. Machine B makes the rest.
80% of the bolts from machine A are faulty.
30% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = $0.4 \times 0.8 + 0.6 \times 0.3 = 0.32 + 0.18 = 0.5$
(b) P(A and faulty) = $0.4 \times 0.8 = 0.32$
P(A | faulty) = $0.32 \div 0.5 = \frac{16}{25}$
Answer: (a) 0.5 (b) $\frac{16}{25}$

10 Loci and regions

ABCD is a rectangle with AB = 14 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 10^2 = \frac{25}{2}\pi$ cm^2
Answer: $\frac{25}{2}\pi$ cm^2