Day 358 **S Tier 10-a-day: Model Solutions**

1 Pressure

A stone block in the shape of a cuboid weighs 1200 N.
Its smallest face has area 150 cm^2. Its largest face has area 600 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the stone block can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 600 cm^2 = 0.06 m^2
(a) Greatest pressure is on the smallest face: 1200 ÷ 0.015 = 80,000 N/m^2
(b) Least pressure is on the largest face: 1200 ÷ 0.06 = 20,000 N/m^2
Answer: (a) 80,000 N/m^2 (b) 20,000 N/m^2

2 Comparing costs

Zip Taxis has a fixed charge of £1.50 plus £4.50 per mile.
Go Cabs has a fixed charge of £28.50 plus £1.50 per mile.
(a) Work out the number of miles for which the two costs are the same.
(b) Which is cheaper for 15 miles, and by how much?

(a) Let n be the number of miles. Working in pounds, the costs are equal when 1.50 + 4.50n = 28.50 + 1.50n
3n = 27, so n = 9
(b) Zip Taxis: £1.50 + £4.50 × 15 = £69
Go Cabs: £28.50 + £1.50 × 15 = £51
Go Cabs is cheaper by £18
Answer: (a) 9 (b) Go Cabs, by £18

3 Arc length and sector area

A sector of a circle has radius 13 cm and angle 216°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{216}{360}$

Arc length = $\frac{216}{360} \times 2 \times \pi \times 13 = \frac{78}{5}\pi$ cm

(b) Area = $\frac{216}{360} \times \pi \times 13^2 = \frac{507}{5}\pi$ cm^2

Answer: (a) $\frac{78}{5}\pi$ cm (b) $\frac{507}{5}\pi$ cm^2

4 Percentages

A is 25% more than S.
S is 15% less than D.
Work out A as a percentage of D.

A = 1.25 × S and S = 0.85 × D
A = 1.25 × 0.85 × D = 1.0625 × D
1.0625 × 100 = 106.25%
Answer: 106.25%

5 Hollow cylinder and density

A metal pipe is a hollow cylinder of length 20 cm. (Calculator)
The outer radius is 5 cm and the inner radius is 3.5 cm.
The density of the metal is 8.5 g/cm^3.
Work out the mass of the pipe. Give your answer in kilograms correct to 3 significant figures.

Volume of metal = $\pi \times 5^2 \times 20 - \pi \times 3.5^2 \times 20 = \pi(25 - 12.25) \times 20$
= 801.11 cm^3
Mass = density × volume = 8.5 × 801.11 = 6809.4 g
= 6.81 kg (3 s.f.)
Answer: 6.81 kg

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6 Tree diagrams without replacement

A bag contains 2 orange sweets, 5 purple sweets and 4 pink sweets.
Two sweets are taken at random without replacement.
Work out the probability that (a) at least one is pink, (b) exactly one is pink.

(a) P(at least one pink) = $1 - \text{P(none)} = 1 - \frac{7}{11} \times \frac{6}{10} = \frac{34}{55}$

(b) P(exactly one pink) = $\frac{4}{11} \times \frac{7}{10} + \frac{7}{11} \times \frac{4}{10} = \frac{28}{55}$

Answer: (a) $\frac{34}{55}$ (b) $\frac{28}{55}$

7 Fractional indices

Given that $x > 0$, solve $x^{\frac{2}{3}} = 9$

Raise both sides to the power $\frac{3}{2}$

$x = (9)^{\frac{3}{2}}$
$x = 27$
Answer: $x = 27$

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 40$
$y = -2x + 10$

Substitute into the first equation: $x^2 + (-2x + 10)^2 = 40$
Expand: $5x^2 - 40x + 60 = 0$
Divide by 5: $x^2 - 8x + 12 = 0$
Factorise: $(x - 2)(x - 6) = 0$, so $x = 2$ or $x = 6$
$y = -2x + 10$ gives $y = 6$ and $y = -2$
Answer: $x = 2, y = 6$ and $x = 6, y = -2$

9 Loci and regions

ABCD is a rectangle with AB = 6 cm and AD = 7 cm.
A point P inside the rectangle is such that AP $\leq$ 2 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP $\leq$ 2 cm: P is inside the quarter circle of radius 2 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 2^2 = \frac{1}{2}\pi$ cm^2

Answer: $\frac{1}{2}\pi$ cm^2

10 Tangency conditions

The line $y = 3x + k$ is a tangent to the curve $y = 3x^2 - x + 4$.
Find the value of k.

Equate: $3x^2 - x + 4 = 3x + k$
Rearrange: $3x^2 - 4x + 4 - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $16 - 4 \times 3 \times (4 - k) = 0$
$12k - 32 = 0$
$k = \frac{8}{3}$

Answer: $k = \frac{8}{3}$