Day 357 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 23 is taken. Work out how many come from Year 9.
(b) A different stratified sample has 10 from Year 8. Work out the size of this sample.

Total = 400 + 250 + 150 + 350 = 1150

(a) Year 9: $\frac{150}{1150} \times 23 = 3$

(b) Fraction sampled = $10 \div 250 = \frac{1}{25}$, so sample size = $\frac{1}{25} \times 1150 = 46$

Answer: (a) 3 (b) 46

2 Fractions

Work out $(4\frac{3}{4} + 3\frac{1}{2}) \div 2\frac{5}{8}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{19}{4} + \frac{7}{2} = \frac{33}{4}$

$\frac{33}{4} \div \frac{21}{8} = \frac{33}{4} \times \frac{8}{21} = \frac{22}{7}$

$= 3\frac{1}{7}$

Answer: $3\frac{1}{7}$

3 Distance-time graphs

The graph shows Leila's cycle ride to a café and back.
(a) Work out Leila's speed on the way back.
(b) Work out Leila's average speed for the whole ride, including the stop.

(a) Return: 16 km in 2 hours, so speed = 16 ÷ 2 = 8 km/h
(b) Total distance = 2 × 16 = 32 km, total time = 4 hours
Average speed = 32 ÷ 4 = 8 km/h
Answer: (a) 8 km/h (b) 8 km/h

4 Arc length and sector area

A sector of a circle has radius 4 cm and angle 150°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{150}{360}$

Arc length = $\frac{150}{360} \times 2 \times \pi \times 4 = \frac{10}{3}\pi$ cm

(b) Area = $\frac{150}{360} \times \pi \times 4^2 = \frac{20}{3}\pi$ cm^2

Answer: (a) $\frac{10}{3}\pi$ cm (b) $\frac{20}{3}\pi$ cm^2

5 Tree diagrams without replacement

A bag contains 4 yellow cubes, 4 white cubes and 3 black cubes.
Two cubes are taken at random without replacement.
Work out the probability that (a) exactly one is yellow, (b) at least one is yellow.

(a) P(exactly one yellow) = $\frac{4}{11} \times \frac{7}{10} + \frac{7}{11} \times \frac{4}{10} = \frac{28}{55}$

(b) P(at least one yellow) = 1 − P(none) = $1 - \frac{7}{11} \times \frac{6}{10} = \frac{34}{55}$

Answer: (a) $\frac{28}{55}$ (b) $\frac{34}{55}$

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6 Prime factors and indices

Write $20^3 \times 12^2$ as a product of powers of prime numbers.

$20 = 2^2 \times 5$, $12 = 2^2 \times 3$

$20^3 = 2^6 \times 5^3$

$12^2 = 2^4 \times 3^2$

Add the powers of each prime: $2^{10} \times 3^2 \times 5^3$

Answer: $2^{10} \times 3^2 \times 5^3$

7 Trigonometric equations

Solve $5 \tan x° = 5$ for $0 \leq x \leq 360$

Rearrange: $5 \tan x° = 5$, so $\tan x° = 1$

The acute angle with tangent 1 is 45°; tangent is positive in the first and third quadrants

$x = 45$ or $x = 225$ (45 and 180 + 45)

Answer: x = 45, 225

8 Simple and compound interest

Gareth invests £1,000 for 4 years at 3.5% per year simple interest.
Amir invests £1,000 for 4 years at 3.5% per year compound interest.
Work out how much more interest Amir earns than Gareth.
Give your answer to the nearest penny. (Calculator)

Simple interest: £1,000 × 3.5% × 4 = £140

Compound: £1,000 × 1.035^4 = £1,147.52

Compound interest = £1,147.52 − £1,000 = £147.52

Difference = £147.52 − £140 = £7.52

Answer: £7.52

9 Equations in disguise

Solve $x - 11\sqrt{x} + 28 = 0$

Let $u = \sqrt{x}$: $u^2 - 11u + 28 = 0$

Factorise: $(u - 7)(u - 4) = 0$, so $u = 7$ or $u = 4$

$\sqrt{x} = 7$ or $\sqrt{x} = 4$

$x = 49$ or $x = 16$

Answer: x = 16 or x = 49

10 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 6) cm, DE = 3 cm and BC = 9 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 6} = \frac{3}{9}$

$9x = 3(2x + 6)$

$9x = 6x + 18$

$3x = 18$

$x = 6$

(b) AB = 2x + 6 = 18 cm

Answer: (a) x = 6 (b) 18 cm