Day 356 **S Tier 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x + 5)°$, $(x + 10)°$, $(x - 25)°$, $(x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$5x + 15 = 540$, so $5x = 525$
$x = 105$
(b) The angles are 105°, 110°, 115°, 80°, 130°
The largest is 130°
Answer: (a) x = 105 (b) 130°

2 Tests of divisibility

The 5-digit number 2239k is a multiple of 4.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 4 if its last two digits form a multiple of 4
Last two digits: 9k
Multiples of 4 of this form: 92, 96
Answer: k = 2 or 6

3 Comparing costs

Sparkle Cleaners has a call-out fee of £6 plus £4.50 per hour.
Fresh Start has a call-out fee of £30.50 plus £1 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 10 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $6 + 4.50n = 30.50 + n$
$3.50n = 24.50$, so $n = 7$
(b) Sparkle Cleaners: £6 + £4.50 × 10 = £51
Fresh Start: £30.50 + £1 × 10 = £40.50
Fresh Start is cheaper by £10.50
Answer: (a) 7 (b) Fresh Start, by £10.50

4 Perimeter and area of a running track

The diagram shows a running track made from a rectangle and two semicircles.
(a) Work out the total length of the boundary of the track.
(b) Work out the area enclosed by the track. Give your answers in terms of π.
The diagram is not drawn accurately.

(a) The two semicircles make one full circle of radius 15: circumference = $2 \times \pi \times 15 = 30\pi$
The two straight sides: $2 \times 60 = 120$
Boundary = $(120 + 30\pi)$ m
(b) The rectangle is 60 m by $2 \times 15 = 30$ m, so its area is $60 \times 30 = 1800$. Circle: $\pi \times 15^2 = 225\pi$
Area = $(1800 + 225\pi)$ m^2
Answer: (a) $(120 + 30\pi)$ m (b) $(1800 + 225\pi)$ m^2

5 Speed-time graphs

The graph shows the speed of a car during 21 seconds.
(a) Work out the acceleration during the first 6 seconds.
(b) Work out the average speed over the whole 21 seconds.

(a) Acceleration = change in speed ÷ time = $(24 - 6) \div 6 = 3$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{6 + 24}{2} \times 6 = 90$; rectangle: $24 \times 9 = 216$; triangle: $\frac{1}{2} \times 6 \times 24 = 72$
Total = 378 m, so average speed = $378 \div 21 = 18$ m/s
Answer: (a) 3 m/s^2 (b) 18 m/s

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6 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 10x + 21}{x^2 + 5x} \times \frac{x^2 - 4x}{x + 7}$

Factorise: $\frac{(x + 7)(x + 3)}{x(x + 5)} \times \frac{x(x - 4)}{(x + 7)}$

Cancel $(x + 7)$ and x

$= \frac{(x - 4)(x + 3)}{(x + 5)}$

Answer: $\mathbf{\frac{(x - 4)(x + 3)}{(x + 5)}}$

7 Index equations

Solve $3^{3x + 1} = 9^{2x + 1}$

Write both sides as powers of 3: $9 = 3^2$

Equate the powers: $3x + 1 = 4x + 2$

$-x = 1$, so $x = -1$

Answer: $x = -1$

8 Line and curve intersections

Solve the simultaneous equations

$y = x^2 + 4x - 4$

$y = 5x - 4$

Substitute: $x^2 + 4x - 4 = 5x - 4$

Rearrange: $x^2 - x = 0$

Factorise: $x(x - 1) = 0$, so $x = 0$ or $x = 1$

When $x = 0$: $y = 5 \times 0 - 4 = -4$

When $x = 1$: $y = 5 \times 1 - 4 = 1$

Answer: $x = 0$, $y = -4$ and $x = 1$, $y = 1$

9 Percentages with an unknown rate

A population is decreased by x% in one year and then decreased by a further x% the next year.
Over the two years the population decreases by 75%.
Work out the value of x.

Yearly multiplier $= 1 - \frac{x}{100}$

Two years: $(1 - \frac{x}{100})^2 = 0.25$

Take the square root: $1 - \frac{x}{100} = 0.5$

$\frac{x}{100} = 0.5$, so $x = 50$

Answer: $x = 50$

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10 Histograms

The histogram shows the lengths of leaves collected from a tree.
(a) Work out the total number of leaves.
(b) Estimate the number of leaves with a length between 20 cm and 55 cm.

Frequency = frequency density × class width
10 to 30: 1.4 × 20 = 28
30 to 50: 2 × 20 = 40
50 to 60: 0.6 × 10 = 6
60 to 70: 0.8 × 10 = 8
(a) Total = 82
(b) 20 to 30: 1.4 × 10 = 14
Whole bar between: 40
50 to 55: 0.6 × 5 = 3, so estimate = 57
Answer: (a) 82 (b) 57