Day 355 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 34 is taken. Work out how many come from Group C.
(b) A different stratified sample has 20 from Group A. Work out the size of this sample.

Total = 250 + 225 + 75 + 300 = 850

(a) Group C: $\frac{75}{850} \times 34 = 3$

(b) Fraction sampled = 20 ÷ 250 = $\frac{2}{25}$, so sample size = $\frac{2}{25} \times 850 = 68$

Answer: (a) 3 (b) 68

2 Fractions

Work out $2\frac{3}{4} \times 2\frac{2}{5} - 1\frac{1}{5}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{11}{4} \times \frac{12}{5} = \frac{33}{5}$

$\frac{33}{5} - \frac{6}{5} = \frac{27}{5}$

$= 5\frac{2}{5}$

Answer: $5\frac{2}{5}$

3 Perpendicular bisector

A is the point (0, 3) and B is the point (6, 5).
Find an equation of the perpendicular bisector of AB. Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = (3, 4)

Gradient of AB = $\frac{1}{3}$

Perpendicular gradient = −3
y − 4 = (−3)(x − 3)
Rearrange to integer coefficients: 3x + y = 13
Answer: 3x + y = 13

4 Quadratic graphs

The curve $y = -x^2 - 8x - 7$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put y = 0. $-x^2 - 8x - 7 = 0$, so $-(x^2 - 8x + 7) = 0$, giving $-(x + 7)(x + 1) = 0$
x + 7 = 0 or x + 1 = 0, so x = −7 or x = −1
(b) The turning point is halfway between A and B: x = (−7 + (−1)) ÷ 2 = −4
When x = −4: $y = -1 \times (-4)^2 - 8 \times (-4) - 7 = 9$
Answer: (a) A(−7, 0), B(−1, 0) (b) (−4, 9)

5 Bounds of speed, density and pressure

A runner covers a distance of 298 m, correct to 3 significant figures.
The race takes 84 s, correct to the nearest whole number.
Work out the upper bound of the speed.
Give your answer correct to 3 significant figures. (Calculator)

297.5 ≤ distance < 298.5; 83.5 ≤ time < 84.5
Upper bound of speed = biggest distance ÷ smallest time = 298.5 ÷ 83.5
= 3.57485... = 3.57 (3 s.f.)
Answer: 3.57 m/s

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6 Tree diagrams and reverse probability

The probability that it rains on a day is 0.6.
If it rains, the probability that Ali is late for school is 0.2.
If it does not rain, the probability that Ali is late is 0.4.
(a) Work out the probability that Ali is late.
(b) Ali is late. Work out the probability that it rained.

(a) P(late) = 0.6 × 0.2 + 0.4 × 0.4 = 0.12 + 0.16 = 0.28
(b) P(rain and late) = 0.12

P(rain | late) = 0.12 ÷ 0.28 = $\frac{3}{7}$

Answer: (a) 0.28 (b) $\frac{3}{7}$

7 Terms of a quadratic sequence

The nth term of a sequence is $2n^2 + 6n + 1$.
(a) Work out the 10th term.
(b) Is 610 a term of the sequence? You must show your working.

(a) $2 \times 10^2 + 6 \times 10 + 1 = 261$
(b) Solve $2n^2 + 6n + 1 = 610$, so $2n^2 + 6n - 609 = 0$
$b^2 - 4ac = 6^2 - 4 \times 2 \times (-609) = 4908$, which is not a square number
So n is not a whole number, and 610 is not a term
Answer: (a) 261 (b) No

8 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (2, 32) and (4, 128).
Find the value of k and the value of a.

Substitute the points: $k a^2 = 32$ and $k a^4 = 128$
Divide: $a^2 = \frac{128}{32} = 4$
$a = 2$ (positive, since $a > 0$)
From the first point: $k \times 4 = 32$, so $k = 8$
Answer: k = 8, a = 2

9 Mixtures and percentage profit

A shop mixes two types of tea.
Type A costs £5.00 per kg and type B costs £14.00 per kg.
They are mixed in the ratio A : B = 4 : 5.
The mixture is sold for £11.00 per kg.
Work out the percentage profit.

Cost of 9 kg of mixture = 4 × £5.00 + 5 × £14.00 = £90.00
Cost per kg = £90.00 ÷ 9 = £10.00
Profit per kg = £11.00 − £10.00 = £1.00
Percentage profit = 1.00 ÷ 10.00 × 100 = 10%
Answer: 10%

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10 Angles of elevation

A point A is on level ground in line with the base of a mast. (Calculator)
The angle of elevation of the top of the mast from A is 53°.
B is a point on the same line, 60 m further from the mast than A.
The angle of elevation of the top of the mast from B is 24°.
Work out the height of the mast. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 53° = \frac{h}{x}$, so $x = h \div \tan 53°$

From B: $\tan 24° = \frac{h}{x + 60}$, so $x + 60 = h \div \tan 24°$

Subtract: $60 = h \div \tan 24° - h \div \tan 53° = h(2.2460 - 0.75355)$
$h = 60 \div 1.4925 = 40.2$ m (1 d.p.)
Answer: 40.2 m