Day 354 **S Tier 10-a-day: Model Solutions**

1 Equation of a curve from a sketch

The diagram shows a sketch of a quadratic curve with a turning point at $(-4, -8)$.
The curve crosses the y-axis at (0, 40).
Find an equation of the curve.
Give your answer in the form $y = ax^2 + bx + c$.

The turning point is $(-4, -8)$, so $y = a(x + 4)^2 - 8$
When x = 0, y = 40: $a \times 16 - 8 = 40$, so a = 3
$y = 3(x + 4)^2 - 8$
Expand: $y = 3x^2 + 24x + 40$
Answer: $y = 3x^2 + 24x + 40$

2 Weighted percentages

The ratio of juniors to seniors in a school is 4 : 1.
65% of the juniors and 30% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 80 juniors and 20 seniors
65% of 80 = 52 and 30% of 20 = 6
Together: 58 out of 100
$58 \div 100 \times 100 = 58\%$
Answer: 58%

3 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 2. N is on AB with AN : NB = 2 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) $AB = -\mathbf{a} + \mathbf{b}$, so $AN = \frac{2}{5}(\mathbf{b} - \mathbf{a})$

$ON = OA + AN = \mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a}) = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

(b) $OM = \frac{1}{3}\mathbf{a}$

$MN = ON - OM = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \frac{1}{3}\mathbf{a} = \frac{4}{15}\mathbf{a} + \frac{2}{5}\mathbf{b}$

Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ (b) $\frac{4}{15}\mathbf{a} + \frac{2}{5}\mathbf{b}$

4 Upper and lower bounds

a = 6.1 and b = 4,
both correct to 1 decimal place.
Work out the lower bound of a × b.

Each value is rounded, so it can be up to 0.05 above or below
$6.05 \leq a < 6.15$; $3.95 \leq b < 4.05$
Lower bound = $6.05 \times 3.95 = 23.8975$
Answer: 23.8975

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5 Column vectors

$\mathbf{a} = \begin{pmatrix}1\\5\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}2\\-2\end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix}5\\1\end{pmatrix}$

$\mathbf{a} + k\mathbf{b}$ is parallel to $\mathbf{c}$.

Work out the value of k.

$\mathbf{a} + k\mathbf{b} = \begin{pmatrix}1 + 2k\\5 - 2k\end{pmatrix}$

Parallel to $\mathbf{c}$: $\frac{1 + 2k}{5} = \frac{5 - 2k}{1}$

$1(1 + 2k) = 5(5 - 2k)$

$k \times (12) = 24$

$k = 2$

Answer: $\mathbf{k = 2}$

6 Percentages of percentages

In a group of employees, 30% are men and the rest are women.

25% of the men and 15% of the women work from home.

What percentage of all the employees work from home?

Take 100 employees: 30 are men and 70 are women

25% of 30 = 7.5 and 15% of 70 = 10.5

7.5 + 10.5 = 18 out of 100

Answer: 18%

7 Winning a match

Jamal and Zane play a match. The first to win 2 table tennis games wins the match.

The probability that Jamal wins any one of the table tennis games is $\frac{3}{5}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Jamal wins then loses) = $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

P(Jamal loses then wins) = $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

Total = $2 \times \frac{6}{25} = \frac{12}{25}$

Answer: $\frac{12}{25}$

8 Rearranging when the subject appears twice

Make r the subject of $P = \frac{4r + 1}{r - 4}$

Multiply by $r - 4$: $P(r - 4) = 4r + 1$

Expand: $rP - 4P = 4r + 1$

Collect the r terms: $rP - 4r = 1 + 4P$

Factorise: $r(P - 4) = 4P + 1$

$r = \frac{4P + 1}{P - 4}$

Answer: $\mathbf{r = \frac{4P + 1}{P - 4}}$

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9 Income tax

In one country, income tax is worked out like this:
The first £15,000 of income is not taxed.
The next £37,500 is taxed at 20%.
All income above that is taxed at 40%.
Amir pays £14,100 in income tax in a year.
Work out Amir's income for the year.

Tax on the 20% band = 20% of £37,500 = £7,500
Tax paid at 40% = £14,100 − £7,500 = £6,600
Income taxed at 40% = £6,600 ÷ 0.4 = £16,500
Income = £15,000 + £37,500 + £16,500 = £69,000
Answer: £69,000

10 Probability from a frequency tree

In a population of 10,000 people, 2% have a certain disease.
A test gives a positive result for 98% of people who have the disease and for 2% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

2% of 10,000 = 200 have the disease and 9,800 do not
Flagged and genuine: 200 × 98% = 196
Flagged but not: 9,800 × 2% = 196
Total flagged or positive = 196 + 196 = 392
Probability = 196 ÷ 392 = $\frac{1}{2}$

Answer: $\frac{1}{2}$