Day 353 **S Tier 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 29 is taken. Work out how many come from Group B.
(b) A different stratified sample has 30 from Group B. Work out the size of this sample.

Total = 100 + 200 + 80 + 200 = 580

(a) Group B: $\frac{200}{580} \times 29 = 10$

(b) Fraction sampled = $30 \div 200 = \frac{3}{20}$, so sample size = $\frac{3}{20} \times 580 = 87$

Answer: (a) 10 (b) 87

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(4x - 10)°$
Angle PQR = $(3x + 30)°$
Angle PRS = $(6x + 30)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
$6x + 30 = 7x + 20$
$x = 10$
(b) Angle PRS = 90° so angle PRQ = $180° - 90° = 90°$
Answer: (a) x = 10 (b) 90°

3 Fractions

Work out $(2\frac{1}{2} - 2\frac{2}{5}) \times \frac{2}{3}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions
$\frac{5}{2} - \frac{12}{5} = \frac{1}{10}$
$\frac{1}{10} \times \frac{2}{3} = \frac{1}{15}$
Answer: $\frac{1}{15}$

4 Arc length and sector area

A sector of a circle has radius 4 cm and angle 72°.
(a) Work out the length of the arc of the sector.
(b) Work out the area of the sector. Give each answer in terms of π.

(a) Fraction of the circle = $\frac{72}{360}$

Arc length = $\frac{72}{360} \times 2 \times \pi \times 4 = \frac{8}{5}\pi$ cm

(b) Area = $\frac{72}{360} \times \pi \times 4^2 = \frac{16}{5}\pi$ cm^2

Answer: (a) $\frac{8}{5}\pi$ cm (b) $\frac{16}{5}\pi$ cm^2

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5 Recurring decimals

Write $0.9\dot{1}$ as a fraction in its simplest form.

Let x = 0.9111...
10x = 9.1111...
100x = 91.1111...
Subtract: 90x = 82
$x = \frac{82}{90} = \frac{41}{45}$

Answer: $\mathbf{\frac{41}{45}}$

6 Composite functions: fg and gf

$f(x) = 4x + 5$ and $g(x) = x^2 + 1$
Solve fg(x) = gf(x)

$fg(x) = f(x^2 + 1) = 4x^2 + 9$
$gf(x) = g(4x + 5) = (4x + 5)^2 + 1 = 16x^2 + 40x + 26$
Set them equal and rearrange: $12x^2 + 40x + 17 = 0$
Factorise: $(2x + 1)(6x + 17) = 0$
$x = -\frac{17}{6}$ or $x = -\frac{1}{2}$

Answer: $\mathbf{x = -\frac{17}{6}}$ **or** $\mathbf{x = -\frac{1}{2}}$

7 Surds

Given that $\sqrt{27} - \sqrt{3} = \sqrt{n}$,
find the value of n.

$\sqrt{27} = 3\sqrt{3}$
$3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$
$2\sqrt{3} = \sqrt{12}$, so n = 12
Answer: n = 12

8 Cone made from a sector

A sector of a circle has radius 12 cm and angle 240°.
The sector is made into a cone by joining the two straight edges together.
(a) Work out the radius of the base of the cone.
(b) Work out the exact volume of the cone. Give your answer in terms of π, as a surd if necessary.

(a) Arc length of the sector = $\frac{240}{360} \times 2\pi \times 12 = 16\pi$ cm
This is the circumference of the base: $2\pi r = 16\pi$, so r = 8 cm
(b) Slant height = 12 cm, so height$^2 = 12^2 - 8^2 = 80$, height = $4\sqrt{5}$ cm
Volume = $\frac{1}{3} \times \pi \times 8^2 \times 4\sqrt{5} = \frac{256\sqrt{5}}{3}\pi$ cm^3

Answer: (a) 8 cm (b) $\mathbf{\frac{256\sqrt{5}}{3}\pi}$ **cm**^{3}

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9 Inverse proportion

T is inversely proportional to the square of n.
When n = 4, T = 36.
Find the value of T when n = 6.

$T = \frac{k}{n^2}$

$36 = \frac{k}{16}$, so $k = 36 \times 16 = 576$

$T = \frac{576}{36} = 16$

Answer: T = 16

10 Equal roots and real roots

Show that the equation $x^2 + kx - 5 = 0$ has two distinct real solutions for all values of k.

$b^2 - 4ac = k^2 - 4 \times 1 \times (-5)$

$= k^2 + 20$

k^2 is never negative, so $k^2 + 20$ is always at least 20, which is greater than 0

So $b^2 - 4ac > 0$ for all k, and there are always two distinct real solutions

Answer: Shown