Day 352 **S Tier 10-a-day: Model Solutions**

1 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = (4x + 10)°
Angle ABC = (x + 4)°
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
(4x + 10) + 2(x + 4) = 180
6x + 18 = 180, so 6x = 162
x = 27
(b) Angle BAC = 4 × 27 + 10 = 118°
Answer: (a) x = 27 (b) 118°

2 Winning a match

Kira and Sofia play a match. The first to win 2 chess games wins the match.

The probability that Kira wins any one of the chess games is $\frac{4}{5}$.

Work out the probability that Kira wins the match.

Kira wins by WW (2 games) or WLW or LWW (3 games), where W means Kira wins that game

P(WW) = $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$

P(WLW) = P(LWW) = $\frac{4}{5} \times \frac{1}{5} \times \frac{4}{5} = \frac{16}{125}$

Total = $\frac{16}{25} + 2 \times \frac{16}{125} = \frac{112}{125}$

Answer: $\frac{112}{125}$

3 Factorising fully

Factorise fully $5x^2 + 15x - 50$

Take out the common factor 5: $5(x^2 + 3x - 10)$
Factorise the quadratic: $5(x - 2)(x + 5)$
Answer: $5(x - 2)(x + 5)$

4 Multiplying and dividing surds

Simplify fully $\frac{4\sqrt{90}}{3\sqrt{10}}$

$\sqrt{90} = 3\sqrt{10}$
$\frac{4\sqrt{90}}{3\sqrt{10}} = \frac{12\sqrt{10}}{3\sqrt{10}}$
$= 4$
Answer: 4

5 Trigonometric equations

Solve $2 \sin x° - 4 = -2$ for $0 \leq x \leq 360$

Rearrange: 2 sin x° = 2, so sin x° = 1
From the graph of y = sin x°, sin x° = 1 when x = 90
Answer: x = 90

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6 Comparing box plots

The box plots show the test marks of the students in two classes, A and B.
(a) Work out the interquartile range for each.
(b) Compare the distributions of A and B.

(a) IQR of A = 60 − 25 = 35
IQR of B = 70 − 25 = 45
(b) Class A has the higher median (45 compared with 35), so on average Class A scored higher
Class A has the smaller interquartile range (35 compared with 45), so its marks are more consistent
Answer: (a) A: 35, B: 45 (b) A has the higher median, A has the smaller IQR

7 LCM problems

A red light flashes every 4 minutes.
A blue light flashes every 8 minutes.
A green light flashes every 24 minutes.
They all flash together at 6:00 am.
(a) At what time do they next flash together?

Find the LCM of 4, 8 and 24
$4 = 2^2$, $8 = 2^3$, $24 = 2^3 \times 3$
LCM = $2^3 \times 3 = 24$
24 minutes after 6:00 am is 6:24 am
Answer: 6:24 am

8 Perpendicular bisector

A is the point (0, −2) and B is the point (10, 4).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{0+10}{2}, \frac{-2+4}{2}) = (5, 1)$

Gradient of AB = $\frac{4-(-2)}{10-0} = \frac{3}{5}$

The perpendicular gradient is $-1 \div \frac{3}{5} = -\frac{5}{3}$

Using the midpoint: $y - 1 = -\frac{5}{3}(x - 5)$

Multiply by 3 and rearrange: 5x + 3y = 28
Answer: 5x + 3y = 28

9 Tangent to a circle

A circle has equation $x^2 + y^2 = 625$.
The point P(7, −24) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{24}{7}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{24}{7}) = \frac{7}{24}$

$y - (-24) = \frac{7}{24}(x - 7)$

Multiply out and rearrange: $7x - 24y = 625$
(b) On the x-axis y = 0, so 7x = 625
$x = \frac{625}{7}$, so T is $(\frac{625}{7}, 0)$

Answer: (a) $7x - 24y = 625$ (b) T = $(\frac{625}{7}, 0)$

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10 Percentage change and proportion

The brightness of a lamp is inversely proportional to the square of the distance from the lamp.
The distance from the lamp is increased by 300%.
Work out the percentage change in the brightness of a lamp.

The distance from the lamp is multiplied by 4

The multiplier for the brightness of a lamp is $1 \div 4^2 = 0.0625$
0.0625 is a decrease of 93.75%
Answer: 93.75% decrease