Day 351 **S Tier 10-a-day: Model Solutions**

1 Fractions

Work out $(3\frac{1}{2} - \frac{1}{2}) \div 2\frac{2}{5}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{7}{2} - \frac{1}{2} = 3$

$3 \div \frac{12}{5} = 3 \times \frac{5}{12} = \frac{5}{4}$

$= 1\frac{1}{4}$

Answer: $1\frac{1}{4}$

2 Arithmetic sequences

The 2nd term of an arithmetic sequence is 2.
The 5th term of the sequence is 26.
(a) Find an expression for the nth term of the sequence.
(b) Is 266 a term of the sequence? You must show your working.

(a) Common difference = $(26 - 2) \div (5 - 2) = 8$
nth term = $8n + c$. For n = 2: $8 \times 2 + c = 2$, so $c = -14$
nth term = $8n - 14$
(b) Solve $8n - 14 = 266$: $8n = 280$, so $n = 35$
n is a positive whole number, so 266 is the 35th term
Answer: (a) $8n - 14$ (b) Yes, it is the 35th term

3 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = $(4x - 10)°$
Angle ABC = $(3x - 40)°$
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
$(4x - 10) + 2(3x - 40) = 180$
$10x - 90 = 180$, so $10x = 270$
$x = 27$
(b) Angle BAC = $4 \times 27 - 10 = 98°$
Answer: (a) x = 27 (b) 98°

4 Standard form

Work out $(8.8 \times 10^5) \div (4 \times 10^6)$
Give your answer in standard form.

$8.8 \div 4 = 2.2$
$10^5 \div 10^6 = 10^{-1}$
$= 2.2 \times 10^{-1}$
Answer: 2.2×10^{-1}

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5 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on AB with AN : NB = 1 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{1}{2}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

(b) OM = $\frac{1}{5}\mathbf{a}$

MN = ON − OM = $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} - \frac{1}{5}\mathbf{a} = \frac{3}{10}\mathbf{a} + \frac{1}{2}\mathbf{b}$

Answer: (a) $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (b) $\frac{3}{10}\mathbf{a} + \frac{1}{2}\mathbf{b}$

6 Prime factors and indices

Write $20^3 \times 4^2$ as a product of powers of prime numbers.

$20 = 2^2 \times 5$, $4 = 2^2$
$20^3 = 2^6 \times 5^3$
$4^2 = 2^4$
Add the powers of each prime: $2^{10} \times 5^3$
Answer: $2^{10} \times 5^3$

7 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.
An estimate for the mean is 20.1.
Work out the value of x.

Mid-points: 12.5, 17.5, 22.5, 27.5
Estimated mean = $\Sigma fm \div \Sigma f$, so $(645 + 22.5x) \div (34 + x) = 20.1$
$645 + 22.5x = 20.1(34 + x) = 683.4 + 20.1x$
$(22.5 - 20.1)x = 683.4 - 645$
$2.4x = 38.4$, so $x = 16$
Answer: x = 16

8 Direct and inverse proportion

T is inversely proportional to the square root of l.
When l = 4, T = 2.
(a) Find a formula for T in terms of l.
(b) Find the value of l when T = 4.

(a) $T = \frac{k}{\sqrt{l}}$

Substitute l = 4 and T = 2: $2 = \frac{k}{\sqrt{4}}$

k = 4

$T = \frac{4}{\sqrt{l}}$

(b) $4 = \frac{4}{\sqrt{l}}$, so $\sqrt{l} = \frac{4}{4} = 1$, so l = 1

Answer: (a) $T = \frac{4}{\sqrt{l}}$ (b) l = 1

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9 At least one

The probability that a light bulb in a hallway fails in any month is $\frac{1}{4}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 5 months.
Give your answer correct to 3 significant figures. (Calculator)

P(never) = $(1 - \frac{1}{4})^5 = \frac{3}{4}^5 = 0.2373046875$

P(at least once) = 1 − P(never) = 1 − 0.2373046875
= 0.763

Answer: 0.763

10 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 85$

$y = -x - 1$

Substitute into the first equation: $x^2 + (-x - 1)^2 = 85$

Expand: $2x^2 + 2x - 84 = 0$

Divide by 2: $x^2 + x - 42 = 0$

Factorise: $(x + 7)(x - 6) = 0$, so $x = -7$ or $x = 6$

$y = -x - 1$ gives $y = 6$ and $y = -7$

Answer: x = −7, y = 6 and x = 6, y = −7