Day 350 **S Tier 10-a-day: Model Solutions**

1 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{16^2 + 15^2 - 13^2}{2 \times 16 \times 15} = \frac{312}{480}$

$\theta = 49.5°$ (1 d.p.)

(b) Area $= \frac{1}{2} \times 16 \times 15 \times \sin 49.5°$

$= 91.2\ \text{cm}^2$ (3 s.f.)

Answer: (a) 49.5° (b) 91.2 cm^2

2 Tests of divisibility

The 5-digit number 7535k is a multiple of 4.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 4 if its last two digits form a multiple of 4
Last two digits: 5k
Multiples of 4 of this form: 52, 56
Answer: k = 2 or 6

3 Trigonometric graphs

The diagram shows the graph of y = a sin bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 90°.

(a) The maximum value is 4, so a = 4
The graph completes 3 full cycles between 0° and 360°, so b = 3
(b) $y = 4 \sin (3 \times 90°) = 4 \sin 270°$
$\sin 270° = -1$, so $y = -4$
Answer: (a) a = 4, b = 3 (b) y = −4

4 Composite functions

$f(x) = -3x + 5$ and $g(x) = 4x^2 - 5$
(a) Find fg(x).
(b) Solve $fg(x) = -88$

(a) $fg(x) = f(4x^2 - 5) = -3(4x^2 - 5) + 5$
$= -12x^2 + 20$
(b) $-12x^2 + 20 = -88$, so $-12x^2 = -108$
$x^2 = 9$, so $x = \pm 3$
Answer: (a) $-12x^2 + 20$ (b) x = ±3

5 Exchange rates

Oscar changes £1000 into dollars at £1 = $1.15.
Oscar spends $630 and then changes the rest back into pounds at £1 = $1.25.
How many pounds does Oscar get back?

£1000 × 1.15 = $1150
1150 − 630 = $520
520 ÷ 1.25 = £416
Answer: £416

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6 Algebraic proof

Prove algebraically that $(6n + 1)^2 - 1$ is a multiple of 6 for all integer values of n.

$(6n + 1)^2 - 1 = 36n^2 + 12n + 1 - 1$

$= 36n^2 + 12n$

$= 6(6n^2 + 2n)$, which is a multiple of 6

Answer: Proved

7 Percentages with an unknown rate

The price of a television is decreased by x%.
The new price is then increased by x%.
The final price is 9% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 9% less, so the multiplier is 0.91: $1 - \frac{x^2}{10000} = 0.91$

$\frac{x^2}{10000} = 0.09$, so $x^2 = 900$ and $x = 30$

Answer: x = 30

8 Functions that are their own inverse

$f(x) = \frac{-5x + 1}{2x + 5}$ where $x \neq -\frac{5}{2}$

(a) Show that ff(x) = x.
(b) Write down $f^{-1}(x)$.

(a) Replace x in f(x) by f(x), then multiply the top and the bottom by (2x + 5)

$ff(x) = \frac{(-5)(-5x + 1) + 1(2x + 5)}{2(-5x + 1) - (-5)(2x + 5)}$

Top = $25x - 5 + 2x + 5 = 27x$

Bottom = $-10x + 2 + 10x + 25 = 27$

$ff(x) = \frac{27x}{27} = x$

(b) ff(x) = x, so f is its own inverse: $f^{-1}(x) = \frac{-5x + 1}{2x + 5}$

Answer: (b) $f^{-1}(x) = \frac{-5x + 1}{2x + 5}$

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to t.
When v = 1 and t = 3, P = 2.
Work out the value of P when v = 6 and t = 4.

$P = \frac{kv^2}{t}$

$2 = \frac{k \times 1}{3}$, so $k = 2 \times 3 \div 1 = 6$

$P = \frac{6 \times 36}{4} = 54$

Answer: P = 54

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10 Bounds and accuracy

a = 27.1 and b = 9.02, both correct to 3 significant figures.
Work out a ÷ b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$27.05 \leq a < 27.15$; $9.015 \leq b < 9.025$
Lower bound = 27.05 ÷ 9.025 = 2.9972...
Upper bound = 27.15 ÷ 9.015 = 3.0116...
Both bounds round to 3.0 to 2 significant figures, but not to 3 significant figures
Answer: 3.0 (2 s.f.)