Day 35 **S Tier 10-a-day: Model Solutions**

1 Tangent to a circle

A circle has equation $x^2 + y^2 = 169$.
The point P(−12, 5) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{5}{12}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{5}{12}) = \frac{12}{5}$

$y - 5 = \frac{12}{5}(x - (-12))$

Multiply out and rearrange: $12x - 5y = -169$
(b) On the x-axis y = 0, so $12x = -169$
$x = -\frac{169}{12}$, so T is $(-\frac{169}{12}, 0)$

Answer: (a) $12x - 5y = -169$ (b) T = $(-\frac{169}{12}, 0)$

2 Factorising fully

Factorise fully $(x + 2)^2 - 16$

This is a difference of two squares: $(x + 2)^2 - 4^2$
$= (x + 2 - 4)(x + 2 + 4)$
$= (x - 2)(x + 6)$
Answer: $(x - 2)(x + 6)$

3 Simplifying surds

Write $\sqrt{45} - \sqrt{5} + \sqrt{180}$ in the form $a\sqrt{5}$
where a is an integer.

$\sqrt{45} = 3\sqrt{5}$; $\sqrt{180} = 6\sqrt{5}$
$(3 - 1 + 6)\sqrt{5} = 8\sqrt{5}$
Answer: $8\sqrt{5}$

4 Average speed

Liam drives 80 km at an average speed of 80 km/h.
The drive then continues for another 30 km at an average speed of 20 km/h.
Work out the average speed for the whole journey.

Time for first part = 80 ÷ 80 = 1 hours
Time for second part = 30 ÷ 20 = 1.5 hours
Total distance = 110 km, total time = 2.5 hours
Average speed = 110 ÷ 2.5 = 44 km/h
Answer: 44 km/h

5 LCM problems

A red light flashes every 4 minutes.
A blue light flashes every 8 minutes.
A green light flashes every 10 minutes.
They all flash together at 7:00 am.
(a) At what time do they next flash together?

Find the LCM of 4, 8 and 10
$4 = 2^2$, $8 = 2^3$, $10 = 2 \times 5$
LCM = $2^3 \times 5 = 40$
40 minutes after 7:00 am is 7:40 am
Answer: 7:40 am

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6 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is even.

(b) Given that at least one number is 3, work out the probability that the product is even.

(a) There are $6 \times 5 = 30$ equally likely outcomes, and 21 of them satisfy: the product is even

Probability = $\frac{7}{10}$

(b) 10 outcomes satisfy: at least one number is 3. Of these, 5 also satisfy: the product is even

Probability = $\frac{1}{2}$

Answer: (a) $\frac{7}{10}$ (b) $\frac{1}{2}$

7 Terms of a quadratic sequence

The nth term of a sequence is $3n^2 - 2n$.

(a) Work out the 8th term.

(b) Is 178 a term of the sequence? You must show your working.

(a) $3 \times 8^2 - 2 \times 8 + 0 = 176$

(b) Solve $3n^2 - 2n = 178$, so $3n^2 - 2n - 178 = 0$

$b^2 - 4ac = (-2)^2 - 4 \times 3 \times (-178) = 2140$, which is not a square number

So n is not a whole number, and 178 is not a term

Answer: (a) 176 (b) No

8 Laws of indices

Write $\frac{9^{n-1} \times 3^{n-2}}{9^n}$ in the form 3^{an+c}

where a and c are integers.

$9 = 3^2$, $3 = 3^1$, $9 = 3^2$

$9^{n-1} = 3^{2(n-1)}$, $3^{n-2} = 3^{n-2}$, $9^n = 3^{2n}$

Exponent: $2(n-1) + (n-2) - 2n = n - 4$

Answer: a = 1, c = −4

9 Arc length and sector area

A sector of a circle has angle 150° and area $\frac{845}{12}\pi$ cm^2.

(a) Work out the radius of the circle.

(b) Work out the length of the arc of the sector. Give your answer to (b) in terms of π.

(a) Area = $\frac{150}{360} \times \pi \times r^2$, so $\frac{150}{360} \times r^2 = \frac{845}{12}$

$r^2 = \frac{845}{12} \times \frac{360}{150} = 169$

r = 13 cm

(b) Arc length = $\frac{150}{360} \times 2 \times \pi \times 13 = \frac{65}{6}\pi$ cm

Answer: (a) 13 cm (b) $\frac{65}{6}\pi$ cm

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10 Tree diagrams and reverse probability

Machine A makes 70% of the bolts in a factory. Machine B makes the rest.
70% of the bolts from machine A are faulty.
30% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.7 × 0.7 + 0.3 × 0.3 = 0.49 + 0.09 = 0.58
(b) P(A and faulty) = 0.7 × 0.7 = 0.49

P(A | faulty) = 0.49 ÷ 0.58 = $\frac{49}{58}$

Answer: (a) 0.58 (b) $\frac{49}{58}$