Day 349 **S Tier 10-a-day: Model Solutions**

1 Surds and fractions

Simplify $\frac{2}{3+\sqrt{7}} + \frac{1}{3-\sqrt{7}}$

Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{7})(3-\sqrt{7}) = 9 - 7 = 2$

Numerator: $2(3-\sqrt{7}) + (3+\sqrt{7})$

$= 9 - \sqrt{7}$

$\frac{9-\sqrt{7}}{2}$

Answer: $\mathbf{\frac{9-\sqrt{7}}{2}}$

2 Laws of indices with algebra

Simplify fully $\frac{3x^3y^5 \times 5x^6y^2}{3x^6y^3}$

Give your answer in the form kx^ay^b

Numbers: $3 \times 5 \div 3 = 5$

Powers of x: $3 + 6 - 6 = 3$

Powers of y: $5 + 2 - 3 = 4$

$= 5x^3y^4$

Answer: $\mathbf{5x^3y^4}$

3 Circle theorems: angle at the centre

A, B and C are points on a circle, centre O. Angle ACB = 21°.

(a) Work out the size of angle AOB.

(b) Work out the size of angle OAB.

The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 21 = 42°$

(b) OA = OB (radii), so triangle OAB is isosceles

Angle OAB = $(180 - 42) \div 2 = 69°$

Answer: (a) 42° (b) 69°

4 Gradient of a chord

A and B are points on the curve $y = x^2 + x - 5$.

The x coordinate of A is 3 and the x coordinate of B is 5.

Work out the gradient of the line AB.

At A: y = 7. At B: y = 25

A is (3, 7) and B is (5, 25)

Gradient = $\frac{25-7}{5-3} = \frac{18}{2} = 9$

Answer: 9

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5 LCM problems

A church bell rings every 4 minutes.
Another bell rings every 15 minutes.
A third bell rings every 24 minutes.
They all ring together at 10:00 am.
How many times do they all ring together between 10:00 am and 3:00 pm, including both 10:00 am and 3:00 pm if they do?

Find the LCM of 4, 15 and 24
$4 = 2^2$, $15 = 3 \times 5$, $24 = 2^3 \times 3$
LCM $= 2^3 \times 3 \times 5 = 120$
Total time from 10:00 am to 3:00 pm = 300 minutes
$300 \div 120 = 2.5$, so there are 2 more times after the start
Including the start: 2 + 1 = 3
Answer: 3

6 Area of a segment

The circle has centre O and radius 10 cm. A and B are points on the circle and angle AOB = 80°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB $= \frac{80}{360} \times \pi \times 10^2 = 69.813$ cm^2
Area of triangle OAB $= \frac{1}{2} \times 10 \times 10 \times \sin 80° = 49.240$ cm^2
Segment = sector − triangle = 69.813 − 49.240 = 20.573
= 20.6 cm^2 (3 s.f.)
Answer: 20.6 cm^2

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 80 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 80 had a mass greater than 75.

(a) Median at cumulative frequency 40, so median = 50
Lower quartile at 20, so LQ = 40. Upper quartile at 60, so UQ = 60
Interquartile range = 60 − 40 = 20
(b) At 75, cumulative frequency = 74
Number greater = 80 − 74 = 6
Answer: (a) median 50, IQR 20 (b) 6

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 74$
$y = x + 2$

Substitute into the first equation: $x^2 + (x + 2)^2 = 74$
Expand: $2x^2 + 4x - 70 = 0$
Divide by 2: $x^2 + 2x - 35 = 0$
Factorise: $(x + 7)(x - 5) = 0$, so $x = -7$ or $x = 5$
$y = x + 2$ gives $y = -5$ and $y = 7$
Answer: $x = -7$, $y = -5$ and $x = 5$, $y = 7$

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9 Bearings with the sine and cosine rules

A walker leaves a camp P and travels 60 km on a bearing of 200° to Q. (Calculator)
It then travels 39 km on a bearing of 100° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 200° − 180° = 020°
Angle PQR = 80° (the difference between the bearings 020° and 100°)
$PR^2 = 60^2 + 39^2 - 2 \times 60 \times 39 \times \cos 80° = 4308.3$, so PR = 65.6 km

(b) $\cos(QPR) = \frac{60^2 + PR^2 - 39^2}{2 \times 60 \times PR}$, so angle QPR = 35.8°

R is anticlockwise of the direction PQ, so the bearing is 200° − 36° = 164°

Answer: (a) 65.6 km (b) 164°

10 Reverse percentages

The price of a coat is increased by 15%.
Later the new price is reduced by 30% in a sale.
The sale price is £338.10.
Work out the price before the increase. (Calculator)

Multipliers: 1.15 and 0.7
Overall multiplier = 1.15 × 0.7 = 0.805
Original × 0.805 = £338.10
Original = £338.10 ÷ 0.805 = £420

Answer: £420