Day 348 **S Tier 10-a-day: Model Solutions**

1 Tests of divisibility

The 5-digit number 4309k is a multiple of 9.
k is a single digit.
Find all the possible values of k.

A number is a multiple of 9 if its digit sum is a multiple of 9
Digit sum = 4 + 3 + 0 + 9 + k = 16 + k
This must be a multiple of 9, so k = 2
Answer: k = 2

2 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 8 cm, DB = 20 cm and the area of triangle ADE is 72 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 8 + 20 = 28 cm

Length scale factor = $28 \div 8 = \frac{7}{2}$

Area scale factor = $(\frac{7}{2})^2 = \frac{49}{4}$

Area of triangle ABC = $72 \times \frac{49}{4} = 882$ cm^2

Trapezium = $882 - 72 = 810$ cm^2
Answer: 810 cm^2

3 Proof with indices

Show that $5^{k+2} - 5^k$ is a multiple of 24 for all integer values of k.

$5^{k+2} = 5^k \times 5^2$
$5^{k+2} - 5^k = 5^k(5^2 - 1)$
$5^2 - 1 = 24$
$= 24 \times 5^k$, which is a multiple of 24
Answer: 24×5^k

4 Capture and recapture

50 beetles in a field are caught, marked and put back.
Later, a second sample of 40 beetles is caught. 8 of these are marked.
(a) Work out an estimate for the number of beetles in a field.
(b) State one assumption you have made.

(a) The proportion marked in the second sample is 8 out of 40

This should match the proportion marked in the whole population: $\frac{8}{40} = \frac{50}{N}$

N = 50 × 40 ÷ 8 = 250
(b) No beetles are born, die or leave between the samples, and the beetles mix evenly
Answer: (a) 250 (b) The population stays the same between samples

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5 Perpendicular lines

The diagram shows the straight line L, which passes through the points A and B.
The line M is perpendicular to L and passes through B.
Find the equation of M in the form y = mx + c.

Gradient of L = $\frac{-6-0}{0-2} = 3$

Perpendicular gradient = $-1 \div 3 = -\frac{1}{3}$

M passes through B(2, 0): $0 = (-\frac{1}{3}) \times 2 + c$, so $c = \frac{2}{3}$

$y = -\frac{1}{3}x + \frac{2}{3}$

Answer: $y = -\frac{1}{3}x + \frac{2}{3}$

6 Rates of work

A tank is filled by a tap in 6 minutes.
A drain can empty the full tank in 12 minutes.
The tap and the drain are both open, and the tank is empty to begin with.
How long does it take for the tank to fill?

In 1 minute: the tap fills $\frac{1}{6}$ and the drain empties $\frac{1}{12}$

Net: $\frac{1}{6} - \frac{1}{12} = \frac{6}{72}$

Time = $\frac{72}{6}$ = 12 minutes

Answer: 12 minutes

7 Perpendicular bisector

A is the point (2, 7) and B is the point (4, −1).
Find an equation of the perpendicular bisector of AB.
Give your answer in the form ax + by = c, where a, b and c are integers.

Midpoint of AB = $(\frac{2+4}{2}, \frac{7+(-1)}{2}) = (3, 3)$

Gradient of AB = $\frac{-1-7}{4-2} = -4$

The perpendicular gradient is $-1 \div (-4) = \frac{1}{4}$

Using the midpoint: $y - 3 = \frac{1}{4}(x - 3)$

Multiply by 4 and rearrange: $-x + 4y = 9$
Multiply by −1: $x - 4y = -9$
Answer: $x - 4y = -9$

8 Exact lengths and areas

A rectangle has area $(17 + 7\sqrt{5})$ cm^2 and length $(2 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{17 + 7\sqrt{5}}{2 + \sqrt{5}}$

Multiply top and bottom by $(2 - \sqrt{5})$:
Numerator: $(17 + 7\sqrt{5})(2 - \sqrt{5}) = -1 - 3\sqrt{5}$
Denominator: $4 - 5 = -1$
Width = $1 + 3\sqrt{5}$
Answer: $1 + 3\sqrt{5}$ cm

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9 Volume and surface area of composite solids

A solid is made from a cone of height 7 cm and a hemisphere. Both have radius 3 cm.
Work out the total volume of the solid. Give your answer in terms of π.

Cone: $\frac{1}{3} \times \pi \times 3^2 \times 7 = 21\pi$

Hemisphere: $\frac{2}{3} \times \pi \times 3^3 = 18\pi$

Total = $21\pi + 18\pi = 39\pi$ cm^3

Answer: 39π cm^3

10 Probability and quadratic equations

A bag contains some counters. 8 of them are green and the rest are white.
Two counters are taken at random without replacement.

The probability that both are green is $\frac{7}{30}$.

Work out how many counters are white.

Let there be n counters altogether

P(both green) = $\frac{8}{n} \times \frac{7}{n-1} = \frac{56}{n(n-1)}$

$\frac{56}{n(n-1)} = \frac{7}{30}$, so $n(n-1) = 240$, giving $n^2 - n - 240 = 0$

$(n-16)(n+15) = 0$, so $n = 16$ (n cannot be negative)

Number of counters that are white = $16 - 8 = 8$

Answer: 8