Day 347 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x-5}{2} + \frac{x-5}{5} = -7$

Multiply every term by 10: $5(x-5) + 2(x-5) = -70$
Expand: $7x - 35 = -70$
$7x = -35$
$x = -5$
Answer: $x = -5$

2 Error intervals

The speed of a cyclist is 4 km/h, correct to 1 significant figure.
Write down the error interval for the speed of a cyclist, using s.

The value has been rounded to the nearest 1
Half of this is 0.5, so go 0.5 below and 0.5 above
$4 - 0.5 = 3.5$ and $4 + 0.5 = 4.5$
$3.5 \leq s < 4.5$
Answer: $3.5 \leq s < 4.5$

3 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 45°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 45 = 90°$
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = $360 - 90 - 90 - 90 = 90°$
Answer: (a) 90° (b) 90°

4 Average speed

Evan drives 30 km at an average speed of 20 km/h.
The drive then continues for another 75 km at an average speed of 75 km/h.
Work out the average speed for the whole journey.

Time for first part = $30 \div 20 = 1.5$ hours
Time for second part = $75 \div 75 = 1$ hour
Total distance = 105 km, total time = 2.5 hours
Average speed = $105 \div 2.5 = 42$ km/h
Answer: 42 km/h

5 Forming and solving quadratic equations

The product of two positive numbers is 130.
The larger number is 3 more than the smaller number.
Let the smaller number be n.
(a) Show that $n^2 + 3n - 130 = 0$
(b) Work out the two numbers.

(a) The numbers are n and n + 3, so $n(n + 3) = 130$
$n^2 + 3n = 130$, so $n^2 + 3n - 130 = 0$
(b) Factorise: $(n - 10)(n + 13) = 0$
$n = 10$ or $n = -13$. n is positive, so $n = 10$
The numbers are 10 and 13
Answer: (b) 10 and 13

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6 Profit and loss

A shop buys a watch for £170.
It adds a 50% mark-up to get the selling price.
In a sale the selling price is reduced by 25%.
Work out the percentage profit or loss on the original cost.

Marked price = £170 × 1.5 = £255
Sale price = £255 × 0.75 = £191.25
Profit = £21.25
Percentage = $\frac{21.25}{170} \times 100 = 12.5\%$

Answer: 12.5% profit

7 Cumulative frequency graphs

The cumulative frequency graph gives information about the masses of 120 apples picked from a tree.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a mass greater than 65.

(a) Median at cumulative frequency 60, so median = 40
Lower quartile at 30, so LQ = 30. Upper quartile at 90, so UQ = 50
Interquartile range = 50 − 30 = 20
(b) At 65, cumulative frequency = 111
Number greater = 120 − 111 = 9
Answer: (a) median 40, IQR 20 (b) 9

8 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the length OM.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) AM = 12 ÷ 2 = 6 cm
$OM^2 = OA^2 - AM^2 = 10^2 - 6^2 = 64$
OM = 8 cm
(b) sin(angle AOM) = $\frac{6}{10}$
Angle AOM = 36.87°, so angle AOB = 2 × 36.87 = 73.7°
Answer: (a) 8 cm (b) 73.7°

9 Squares, cubes and prime factors

Find the smallest positive integer k such that 540 × k is a square number.

$540 = 2^2 \times 3^3 \times 5$
For a square number every power must be a multiple of 2
Multiply by 3 × 5 to get $2^2 \times 3^4 \times 5^2$
k = 15
Answer: k = 15

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10 Winning a match

Dina and Hassan play a match. The first to win 2 chess games wins the match.

The probability that Dina wins any one of the chess games is $\frac{1}{3}$.

Work out the probability that the match needs a third one to decide it.

A third one is needed when each player wins one of the first two

P(Dina wins then loses) = $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

P(Dina loses then wins) = $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

Total = $2 \times \frac{2}{9} = \frac{4}{9}$

Answer: $\frac{4}{9}$