Day 346 **S Tier 10-a-day: Model Solutions**

1 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 2.
85% of the juniors and 40% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 60 students in total: 20 juniors and 40 seniors
85% of 20 = 17 and 40% of 40 = 16
Together: 33 out of 60
33 ÷ 60 × 100 = 55%
Answer: 55%

2 Multiplying and dividing surds

Work out $(5\sqrt{3})^2 - (2\sqrt{3})^2$

$(5\sqrt{3})^2 = 5^2 \times 3 = 75$
$(2\sqrt{3})^2 = 2^2 \times 3 = 12$
$75 - 12 = 63$
Answer: 63

3 Comparing costs

Hire Shop A has a delivery charge of £1.50 plus £6 per day.
Hire Shop B has a delivery charge of £22.50 plus £2.50 per day.
(a) Work out the number of days for which the two costs are the same.
(b) Which is cheaper for 14 days, and by how much?

(a) Let n be the number of days. Working in pounds, the costs are equal when 1.50 + 6n = 22.50 + 2.50n
3.50n = 21, so n = 6
(b) Hire Shop A: £1.50 + £6 × 14 = £85.50
Hire Shop B: £22.50 + £2.50 × 14 = £57.50
Hire Shop B is cheaper by £28
Answer: (a) 6 (b) Hire Shop B, by £28

4 Inverse proportion in context

10 painters can paint a hall in 18 days.
After 6 days, 2 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 10 × 18 = 180 painter-days
Work done = 10 × 6 = 60, so work left = 120
Workers left = 10 − 2 = 8
Time = 120 ÷ 8 = 15 days
Answer: 15 days

5 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 48°} = \frac{16}{\sin 39°}$

$x = \frac{16 \times \sin 48°}{\sin 39°} = 18.894$

x = 18.9 cm (1 d.p.)
Answer: AC = 18.9 cm

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6 Iteration

Use the iteration formula $x_{n+1} = \sqrt{5x_n + 14}$ with $x_0 = 3$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt{5 \times 3 + 14} = 5.38516...$

$x_2 = \sqrt{5 \times x_1 + 14} = 6.39733...$

$x_3 = \sqrt{5 \times x_2 + 14} = 6.78135...$

Rounded: $x_1 = 5.385$, $x_2 = 6.397$, $x_3 = 6.781$

Answer: $x_1 = 5.385$, $x_2 = 6.397$, $x_3 = 6.781$

7 Standard form problems

A library has 2×10^4 books.
Each book has 8.8×10^4 words.
Work out the total number of words in all the books.
Give your answer in standard form.

Total = $(2 \times 10^4) \times (8.8 \times 10^4)$

$2 \times 8.8 = 17.6$ and $10^4 \times 10^4 = 10^8$

$= 1.76 \times 10^9$

Answer: 1.76×10^9 words

8 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 74$
$y = -x + 2$

Substitute into the first equation: $x^2 + (-x + 2)^2 = 74$

Expand: $2x^2 - 4x - 70 = 0$

Divide by 2: $x^2 - 2x - 35 = 0$

Factorise: $(x + 5)(x - 7) = 0$, so $x = -5$ or $x = 7$

$y = -x + 2$ gives $y = 7$ and $y = -5$

Answer: $x = -5$, $y = 7$ and $x = 7$, $y = -5$

9 Probability and quadratic equations

A bag contains some sweets. 9 of them are red and the rest are yellow.
Two sweets are taken at random without replacement.

The probability that both are red is $\frac{6}{11}$.

Work out how many sweets are yellow.

Let there be n sweets altogether

P(both red) = $\frac{9}{n} \times \frac{8}{n-1} = \frac{72}{n(n-1)}$

$\frac{72}{n(n-1)} = \frac{6}{11}$, so $n(n-1) = 132$, giving $n^2 - n - 132 = 0$

$(n - 12)(n + 11) = 0$, so $n = 12$ (n cannot be negative)

Number of sweets that are yellow = $12 - 9 = 3$

Answer: 3

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10 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 39°.
B is a point on the same line, 80 m further from the tower than A.
The angle of elevation of the top of the tower from B is 16°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 39° = \frac{h}{x}$, so $x = h \div \tan 39°$

From B: $\tan 16° = \frac{h}{x + 80}$, so $x + 80 = h \div \tan 16°$

Subtract: $80 = h \div \tan 16° - h \div \tan 39° = h(3.4874 - 1.2349)$
$h = 80 \div 2.2525 = 35.5$ m (1 d.p.)
Answer: 35.5 m