Day 345 **S Tier 10-a-day: Model Solutions**

1 Ratio and expressions

x : y = 5 : 4
Work out the ratio (6x − 6y) : (2x + 2y)
Give your answer in its simplest form.

x : y = 5 : 4, so let x = 5t and y = 4t
6(5t) − 6(4t) = 6t
2(5t) + 2(4t) = 18t
6t : 18t = 1 : 3
Answer: 1 : 3

2 Trigonometric equations

Solve $-3\cos x° + 7 = 10$ for $0 \leq x \leq 360$

Rearrange: $-3\cos x° = 3$, so $\cos x° = -1$
From the graph of $y = \cos x°$, $\cos x° = -1$ when $x = 180$
Answer: x = 180

3 Trigonometric graphs

The diagram shows the graph of y = a sin bx, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when x = 75°.

(a) The maximum value is 4, so a = 4
The graph completes 2 full cycles between 0° and 360°, so b = 2
(b) y = 4 sin (2 × 75°) = 4 sin 150°
sin 150° = $\frac{1}{2}$, so y = 2
Answer: (a) a = 4, b = 2 (b) y = 2

4 Exchange rates

Callum changes £300 into euros at £1 = €1.2.
Callum spends €140 and then changes the rest back into pounds at £1 = €1.25.
How many pounds does Callum get back?

£300 × 1.2 = €360
360 − 140 = €220
220 ÷ 1.25 = £176
Answer: £176

5 Upper and lower bounds

a = 35 and b = 16,
both correct to the nearest whole number.
Work out the lower bound of a ÷ b.
Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.5 above or below
$34.5 \leq a < 35.5$; $15.5 \leq b < 16.5$
Smallest answer: smallest a ÷ biggest b = 34.5 ÷ 16.5
= 2.09091... = 2.09 to 3 significant figures
Answer: 2.09

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6 Angles in polygons

The interior angles of a pentagon are
$x°$, $(x - 25)°$, $(x + 10)°$, $(x + 40)°$, $(2x - 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$6x = 540$, so $6x = 540$
$x = 90$
(b) The angles are 90°, 65°, 100°, 130°, 155°
The largest is 155°
Answer: (a) x = 90 (b) 155°

7 Bounds of area and perimeter

A rectangle has length 10 cm and width 8 cm.
Both measurements are correct to the nearest centimetre.
Nia says that the perimeter of the rectangle could be exactly 38 cm.
Is Nia correct? You must show your working.

$9.5 \leq$ length < 10.5 and $7.5 \leq$ width < 8.5
Lower bound of perimeter $= 2 \times 9.5 + 2 \times 7.5 = 34$
Upper bound of perimeter $= 2 \times 10.5 + 2 \times 8.5 = 38$
So $34 \leq$ perimeter < 38
38 is equal to the upper bound, which cannot be reached, so Nia is not correct
Answer: No, 38 is the upper bound, which cannot be reached

8 Perpendicular bisector

A is the point $(0, -6)$ and B is the point $(10, -2)$.
Find an equation of the perpendicular bisector of AB.
Give your answer in the form $y = mx + c$.

Midpoint of AB $= \left(\frac{0 + 10}{2}, \frac{-6 + (-2)}{2}\right) = (5, -4)$

Gradient of AB $= \frac{-2 - (-6)}{10 - 0} = \frac{2}{5}$

The perpendicular gradient is $-1 \div \frac{2}{5} = -\frac{5}{2}$

Using the midpoint: $y + 4 = -\frac{5}{2}(x - 5)$

$y = -\frac{5}{2}x + \frac{17}{2}$

Answer: $y = -\frac{5}{2}x + \frac{17}{2}$

9 Ratio and money transferred

Fatima and Kira share some money in the ratio 3 : 1.
Fatima gives Kira £28.
The ratio of their money is now 1 : 3.
How much money do they have altogether?

Let the amounts be 3k and 1k, so the total is 4k

Fatima now has $3k - 28$, which is $\frac{1}{4}$ of the total

$4(3k - 28) = 1 \times 4k$
$12k - 112 = 4k$, so $8k = 112$ and $k = 14$
Total $= 4 \times 14 =$ £56
Answer: £56

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10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 9 cm, top radius 2 cm and base radius 4 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+9} = \frac{2}{4}$

$4h = 2(h + 9)$, so $2h = 18$ and $h = 9$

Large cone: $\frac{1}{3} \times \pi \times 4^2 \times 18 = 96\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 9 = 12\pi$

Frustum = $96\pi - 12\pi = 84\pi$ cm^3

Answer: 84π cm^3