Day 344 **S Tier 10-a-day: Model Solutions**

1 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 48°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 48°
Angle COB is an exterior angle of triangle OAC: 48 + 48 = 96°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 96°

2 LCM problems

A red light flashes every 4 minutes.
A blue light flashes every 12 minutes.
A green light flashes every 24 minutes.
They all flash together at 6:00 am.
(a) At what time do they next flash together?

Find the LCM of 4, 12 and 24
$4 = 2^2$, $12 = 2^2 \times 3$, $24 = 2^3 \times 3$
LCM = $2^3 \times 3 = 24$
24 minutes after 6:00 am is 6:24 am
Answer: 6:24 am

3 Pressure

A metal crate in the shape of a cuboid weighs 210 N.
Its smallest face has area 150 cm^2. Its largest face has area 250 cm^2.
Pressure = force ÷ area
Work out (a) the greatest pressure and (b) the least pressure the metal crate can exert on the ground, in N/m^2.

1 m^2 = 10 000 cm^2, so 150 cm^2 = 0.015 m^2 and 250 cm^2 = 0.025 m^2
(a) Greatest pressure is on the smallest face: 210 ÷ 0.015 = 14,000 N/m^2
(b) Least pressure is on the largest face: 210 ÷ 0.025 = 8,400 N/m^2
Answer: (a) 14,000 N/m^2 (b) 8,400 N/m^2

4 Inequalities

(a) Solve $2 - 2x > 8$
(b) n is an integer. Write down all the values of n that satisfy
$-2 \leq 3n + 4 \leq 16$

(a) Subtract 2: $-2x > 6$
Divide by -2 and reverse the inequality sign: $x < -3$
(b) Subtract 4: $-6 \leq 3n \leq 12$
Divide by 3: $-2 \leq n \leq 4$
Integers: $-2, -1, 0, 1, 2, 3, 4$
Answer: (a) $x < -3$ (b) $-2, -1, 0, 1, 2, 3, 4$

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 55° and angle CAQ = 79°.
(a) Work out the size of angle ABC.
(b) Work out the size of angle ACB.
The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 79° (alternate segment theorem)
(b) Angle ACB = angle PAB = 55° (alternate segment theorem)
Answer: (a) 79° (b) 55°

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6 Speed-time graphs

The graph shows the speed of a train during 20 seconds.
(a) Work out the acceleration during the first 4 seconds.
(b) Work out the average speed over the whole 20 seconds.

(a) Acceleration = change in speed ÷ time = $(20 - 4) \div 4 = 4$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{4 + 20}{2} \times 4 = 48$; rectangle: $20 \times 9 = 180$; triangle: $\frac{1}{2} \times 7 \times 20 = 70$
Total = 298 m, so average speed = $298 \div 20 = 14.9$ m/s
Answer: (a) 4 m/s^2 (b) 14.9 m/s

7 Algebraic fractions: multiplying and dividing

Simplify fully $\frac{x^2 + 6x - 7}{x^2 + 2x} \times \frac{x^2 - 5x}{x + 7}$

Factorise: $\frac{(x + 7)(x - 1)}{x(x + 2)} \times \frac{x(x - 5)}{(x + 7)}$
Cancel $(x + 7)$ and x
$= \frac{(x - 5)(x - 1)}{(x + 2)}$
Answer: $\frac{\mathbf{(x - 5)(x - 1)}}{\mathbf{(x + 2)}}$

8 Equation of a circle

A and B are the points (3, −6) and (−9, 0).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{3 + (-9)}{2}, \frac{-6 + 0}{2}) = (-3, -3)$
$r^2 = (3 - (-3))^2 + (-6 - (-3))^2 = 36 + 9 = 45$
$(x + 3)^2 + (y + 3)^2 = 45$
Answer: $(x + 3)^2 + (y + 3)^2 = 45$

9 Bounds of roots

The volume of a cube is 801 cm^3, correct to the nearest cm^3.
Work out the lower bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$800.5 \leq$ volume < 801.5
Lower bound of edge $= \sqrt[3]{800.5}$
$= 9.28511... = 9.29$ (3 s.f.)
Answer: 9.29 cm

10 Ratio and money transferred

Priya and Noah share some money in the ratio 9 : 4.
Priya gives Noah £68.
The ratio of their money is now 1 : 8.
How much money do they have altogether?

Let the amounts be 9k and 4k, so the total is 13k
Priya now has $9k - 68$, which is $\frac{1}{9}$ of the total
$9(9k - 68) = 1 \times 13k$
$81k - 612 = 13k$, so $68k = 612$ and $k = 9$
Total $= 13 \times 9 =$ £117
Answer: £117