Day 343 **S Tier 10-a-day: Model Solutions**

1 Cumulative frequency graphs

The cumulative frequency graph gives information about the times of 120 students to complete a puzzle.
(a) Find an estimate for the median and the interquartile range.
(b) Estimate how many of the 120 had a time greater than 55.

(a) Median at cumulative frequency 60, so median = 50
Lower quartile at 30, so LQ = 40. Upper quartile at 90, so UQ = 60
Interquartile range = 60 − 40 = 20
(b) At 55, cumulative frequency = 75
Number greater = 120 − 75 = 45
Answer: (a) median 50, IQR 20 (b) 45

2 Simultaneous equations

Solve the simultaneous equations
$-5x + 4y = 12$
$-x + 2y = 0$

To match the y terms: (2) × 2 gives $-2x + 4y = 0$
Subtract the equations to eliminate y: $-3x = 12$
$x = -4$
Substitute $x = -4$ into (1): $4y + 20 = 12$
$4y = -8$, so $y = -2$
Answer: $x = -4$, $y = -2$

3 Weighted percentages

The ratio of juniors to seniors in a school is 1 : 4.
15% of the juniors and 80% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 20 juniors and 80 seniors
15% of 20 = 3 and 80% of 80 = 64
Together: 67 out of 100
67 ÷ 100 × 100 = 67%
Answer: 67%

4 Equations with indices

Solve $9^{x-2} = 81^{x+2}$

Write both sides as powers of 3: $(3^2)^{x-2} = (3^4)^{x+2}$
$3^{2x-4} = 3^{4x+8}$
Equate the powers: $2x - 4 = 4x + 8$
$-2x = 12$
$x = -6$
Answer: $x = -6$

5 Bounds in geometry

A runner covers 100 metres, correct to the nearest 10 metres, in a time of 14.9 seconds, correct to 1 decimal place. (Calculator)
Work out the upper bound of the runner's average speed in metres per second. Give your answer correct to 3 significant figures.

Distance is between 95 and 105 metres; time is between 14.85 and 14.95 seconds
Speed = distance ÷ time. For the upper bound use the greatest distance and the smallest time
105 ÷ 14.85 = 7.07 m/s (3 s.f.)
Answer: 7.07 m/s

Day 343 **S Tier 10-a-day: Model Solutions**

6 Tree diagrams without replacement

A bag contains 2 yellow cubes, 2 white cubes and 4 black cubes.
Two cubes are taken at random without replacement.
Work out the probability that (a) at least one is yellow, (b) both are the same colour.

(a) P(at least one yellow) = 1 − P(none) = $1 - \frac{6}{8}\times\frac{5}{7} = \frac{13}{28}$

(b) Add the ways: $\frac{2}{8}\times\frac{1}{7} + \frac{2}{8}\times\frac{1}{7} + \frac{4}{8}\times\frac{3}{7} = \frac{2}{7}$

Answer: (a) $\frac{13}{28}$ (b) $\frac{2}{7}$

7 HCF and LCM

The HCF of two numbers is 6 and their LCM is 264.
One of the numbers is 66.
Work out the other number.

HCF × LCM = product of the two numbers
6 × 264 = 1584
Other number = 1584 ÷ 66 = 24
Answer: 24

8 Quadratic formula and surds

Solve $3x^2 + 7x - 8 = 0$
Give your answers in surd form.

Use the quadratic formula with a = 3, b = 7, c = −8

$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 3 \times (-8)}}{2 \times 3}$

$x = \frac{-7 \pm \sqrt{145}}{6}$

Answer: $x = \frac{-7 \pm \sqrt{145}}{6}$

9 Angles of elevation

A point A is on level ground in line with the base of a lighthouse. (Calculator)
The angle of elevation of the top of the lighthouse from A is 41°.
B is a point on the same line, 30 m further from the lighthouse than A.
The angle of elevation of the top of the lighthouse from B is 21°.
Work out the height of the lighthouse. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 41° = \frac{h}{x}$, so $x = h \div \tan 41°$

From B: $\tan 21° = \frac{h}{x + 30}$, so $x + 30 = h \div \tan 21°$

Subtract: $30 = h \div \tan 21° - h \div \tan 41° = h(2.6051 - 1.1504)$
h = 30 ÷ 1.4547 = 20.6 m (1 d.p.)
Answer: 20.6 m

10 Upper and lower bounds

a = 4.2, b = 2.8 and c = 4.6,
all correct to 1 decimal place.

Work out the upper bound of $\frac{a + b}{c}$.

Give your answer correct to 3 significant figures.

Each value is rounded, so it can be up to 0.05 above or below
4.15 ≤ a < 4.25; 2.75 ≤ b < 2.85; 4.55 ≤ c < 4.65
Biggest answer: (biggest a + biggest b) ÷ smallest c = (4.25 + 2.85) ÷ 4.55
= 1.56044... = 1.56 to 3 significant figures
Answer: 1.56