Day 342 **S Tier 10-a-day: Model Solutions**

1 Equations with fractions

Solve $\frac{x+2}{6} + \frac{x-3}{5} = -1$

Multiply every term by 30: $5(x + 2) + 6(x - 3) = -30$
Expand: $11x - 8 = -30$
$11x = -22$
$x = -2$
Answer: $x = -2$

2 Standard form

Work out $(6.2 \times 10^6) \times (3.4 \times 10^{-5})$
Give your answer in standard form.

$6.2 \times 3.4 = 21.08$
$10^6 \times 10^{-5} = 10^1$
$21.08 \times 10^1 = 2.108 \times 10^2$
Answer: 2.108×10^2

3 Comparing costs

FitLife has a joining fee of £4.50 plus £3 per month.
GymNow has a joining fee of £25.50 plus £1.50 per month.
(a) Work out the number of months for which the two costs are the same.
(b) Which is cheaper for 17 months, and by how much?

(a) Let n be the number of months. Working in pounds, the costs are equal when $4.50 + 3n = 25.50 + 1.50n$
$1.50n = 21$, so $n = 14$
(b) FitLife: £4.50 + £3 × 17 = £55.50
GymNow: £25.50 + £1.50 × 17 = £51
GymNow is cheaper by £4.50
Answer: (a) 14 (b) GymNow, by £4.50

4 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 4 cm, DB = 10 cm and the area of triangle ADE is 96 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 4 + 10 = 14 cm

Length scale factor = $14 \div 4 = \frac{7}{2}$

Area scale factor = $(\frac{7}{2})^2 = \frac{49}{4}$

Area of triangle ABC = $96 \times \frac{49}{4} = 1176$ cm^2

Trapezium = $1176 - 96 = 1080$ cm^2
Answer: 1080 cm^2

5 Expanding brackets with surds

Show that $(6 - 2\sqrt{7})(6 + 2\sqrt{7})$ is an integer.

Use the difference of two squares: $(6 - 2\sqrt{7})(6 + 2\sqrt{7}) = 36 - (2\sqrt{7})^2$
$= 36 - 28 = 8$
Answer: 8

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6 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 3 : 2. N is on AB with AN : NB = 2 : 3.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find MN in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{2}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{2}{5}(\mathbf{b} - \mathbf{a}) = \frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

(b) OM = $\frac{3}{5}\mathbf{a}$

MN = ON − OM = $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b} - \frac{3}{5}\mathbf{a} = \frac{2}{5}\mathbf{b}$

Answer: (a) $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ (b) $\frac{2}{5}\mathbf{b}$

7 Geometric sequences

The 2nd term of a geometric sequence is −162.
The 5th term of the sequence is 4374.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 4374 \div (-162) = -27$
$r = \sqrt[3]{-27} = -3$
(b) The 2nd term is $a \times r^1$: $a \times (-3) = -162$
$a = -162 \div (-3) = 54$
Answer: (a) r = −3 (b) 54

8 Estimating the median from grouped data

The table gives information about 82 lengths.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 18, 34, 42, 62, 82
The median is the value at position 82 ÷ 2 = 41
It is in the class 50 to 70, with 34 values before it, so we need 7 more of the 8 values in this class
Median ≈ $50 + \frac{7}{8} \times 20 = 67.5$

Answer: 67.5

9 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 51°.
B is a point on the same line, 50 m further from the tower than A.
The angle of elevation of the top of the tower from B is 37°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 51° = \frac{h}{x}$, so $x = h \div \tan 51°$

From B: $\tan 37° = \frac{h}{x + 50}$, so $x + 50 = h \div \tan 37°$
Subtract: $50 = h \div \tan 37° - h \div \tan 51° = h(1.3270 - 0.80978)$
h = 50 ÷ 0.51726 = 96.7 m (1 d.p.)
Answer: 96.7 m

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10 Relative speed

Jamal and Dina live 120 km apart.
At 10:00 they set off towards each other on the same road.
Jamal drives at 80 km/h and Dina drives at 40 km/h.
(a) At what time do they meet?
(b) How far from Jamal's home do they meet?

Closing speed = 80 + 40 = 120 km/h
(a) Time = 120 ÷ 120 = 1 hours = 60 minutes, so they meet at 11:00
(b) Distance = 80 × 1 = 80 km
Answer: (a) 11:00 (b) 80 km